Problems in Welfare Economics

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1. a. \( y_1 = ax_1 + bx_2 \)  
   b. \( y_1 = x_1^{1/2} x_2^{1/4} \)  
   c. \( y_1 = \min \left( \frac{x_1}{a}, \frac{x_2}{b} \right) \) 
   d. \( y_1 + y_2 = 2x_1^{1/3} x_2^{1/3} \) 
   e. \( y_1 = \frac{4x_1}{y_2} + \frac{x_2}{y_2} \) 
   f. \( y_1^2 + y_2^2 = 10x \)

where \( y_1 \) and \( y_2 \) are outputs; \( x_1 \) and \( x_2 \) are inputs. For each of the above production functions perform the following tasks.

i. Write the function in an implicit form which recognizes the possibility of inefficient production and uses the convention that inputs are designated by negative quantities, that is \( f(y) \leq 0 \).

ii. Where possible, use the first-order condition that we derived for profit maximization (eq. [1] in class notes) to state all applicable necessary conditions. As price vector use \( \mathbf{p} = (p_1, p_2, r_1, r_2) \) or \( (p_1, r_1, r_2) \)

Where this is not possible, state why not and state how the profit-maximizing firm is to determine input and output quantities.

CE, PO, SW, and PPO

2. Cabbage (C) is produced by one firm and tomatoes (T) are produced by another firm in a two-product economy which uses only labor as an input. There are 32 units of labor available, and labor supply is independent of the wage rate.

Industry production functions are as follows:

\[ C = L_{C}^{1/2} \] and \( T = 4L_{T}^{1/2} \)

Aggregate demands in this economy are generated by identical, homothetic tastes which combine to be well represented by the aggregate utility function

\[ U(C, T) = CT \]

a. Find an equation for the production possibilities frontier.

b. Find the competitive equilibrium.

c. Does the competitive equilibrium lie on the production possibilities frontier? Why would you expect this result?
3. Consider a society with two individuals, A and B, and two goods, X and Y. The total amount of X and Y available is 9 units and 12 units, respectively. Utility functions for A and B are

\[ U_A = X_A Y_A \quad \text{and} \quad U_B = X_B^2 Y_B. \]

a. Set up the Pareto optimality problem for this society and use it to find an equation which describes the contract curve.

b. Suppose that initial endowments are as follows: A has 4 units of X and 4 units of Y, and B has 5 units of X and 8 units of Y. Find the competitive equilibrium.

c. Does the competitive equilibrium distribution of goods lie on the contract curve? Why would you expect this result?

4. Robinson Crusoe is shipwrecked on an unpopulated island and must now organize his economic activity. He has 16 units of capital (salvaged from his ship) which is perfectly mobile between the production of saltwater fish (X) and freshwater fish (Y). His preferences for these two commodities can be represented by the utility function: \( U = XY^{4/3} \). He has decided that he will exert 60 units of labor each week. Weekly production functions are known to be as follows:

\[ X = L_x + K_x \quad \text{and} \quad Y = 10L_y^{1/2}K_y^{1/4}. \]

a. Determine Crusoe's production possibility frontier.

b. Determine the island economy's socially optimal output of X and Y.

c. Graph the results of a and b accurately.

5. Two individuals, A and B, are legally permitted to work only 5 days per week. A must sleep 8 hours per day which leaves him with 80 hours per working week. B has only 75 hours available during the same time period because she must sleep 9 hours per day. For each of these individuals, available time is either allocated to labor, \( L_i \), or leisure, \( H_i \). Each individual acts according to a known utility function which depends only on the quantity of food, \( F_i \), and leisure consumed:

\[ U_a = F_a H_a \quad \text{and} \quad U_b = F_b^2 H_b. \]

There are two food-producing farms, C and D. Because the amount of land in each farm is fixed, weekly production depends only on the amount of labor employed:

\[ F_c = 8L_c^{1/2} \quad \text{and} \quad F_d = 28L_d^{1/2}. \]

a. Farm D is obviously more technologically efficient than C. Will farm C produce a nonzero quantity in the competitive equilibrium? Explain.

b. Find the competitive equilibrium. [Some additional assumptions about the distribution of profits are needed.] Verify that all markets clear.

c. Set up the Pareto optimality problem for this setting, and write the corresponding Lagrangian.

6. In a segregated society, men own all of a productive input, E. The other productive input, capital or K, is jointly owned by men and women. Of the two outputs produced in this economy, women prefer y more so than x, and men have an opposite preference. Suppose that aggregated men and aggregated women (2 separate agents) behave according to the following preference structures:

\[ U_m = x_m^3 y_m \quad \text{and} \quad U_w = x_w^{1/3} y_w. \]

m owns 40 units of E and $16 worth of K (the natural numeraire good). w also owns $16 worth of K.
Production of x and y is accomplished according to the following production functions:

\[ x = E_x^{0.5} K_x^{0.5} \quad \text{and} \quad y = E_y + 2K_y. \]

a. Find the competitive equilibrium.
b. What are your expectations regarding the efficiency of your findings in part (a) and why?
c. To investigate how this society performs under various degrees of segregation, recompute the competitive equilibrium assuming that there are 40 units of E and that \( \alpha \% \) of it is owned by men while the rest belongs to women. What aspects of the equilibrium are altered? Are any parts of your findings surprising?

7. On an isolated island, people produce only textile and corn. Textile is made using only labor, each worker producing 10 yards per period. Corn is grown on the island’s cultivated 1,000 acres using labor as well. A single landlord owns this acreage and does not provide labor. Given available land, the production function for growing corn is

\[ C = 20 \cdot L_C - 0.01 \cdot L_C^2 \]

where \( C \) is corn output per period and \( L_C \) is the number of workers employed in the corn sector during the period.

There are 1,000 workers on the island, and they move freely between the two sectors of the economy. The identical islanders, 1000 workers and 1 landlord, consume both corn and textiles, and they perceive these goods as perfect substitutes. Every islander behaves as if his/her utility function is

\[ U(C,T) = 4C + 4T \]

where \( T \) denotes the amount of textile consumed. All income is expended on corn and textiles.

a. What are prices, outputs, wages, and per-acre rents in the competitive equilibrium?
b. What are the equilibrium bundles consumed by each worker? How much is consumed by the landlord?
c. Plot whatever efficiency frontiers or curves that you can.
d. Is the equilibrium economically efficient? Explain.

8. The 100 people living on a intergalactic craft named V-Forge pass their time in the enjoyment of various forms of Art (all Art is rival and ceases to exist upon consumption). All other, material wants are provided by robots called Hueies. There are two types of people, who differ according to their preferences. Type 1 people are Art lovers with equivalent preferences. Type 2 people are also identical.

\[ U_1 = A_1^k H_1 \quad U_2 = A_2^{1/k} H_2 \quad k \text{ (a single constant)}>1, \]

where \( A_i \) denotes composite Art consumption by person \( i \) and \( H_i \) is the number of Hueies operating as the person's servants. There are 50 people of each type. Each and every person has one Huey; these can either be used as servants or they can be leased out. A Huey's time is divisible.

There is a single business aboard the spaceship, and this business is the only source of Art. The business is run by a computer. The computer is capable of original thought, and it rents Hueies from people and assigns them original Art production tasks. The business's production function is \( A = H^{1/2} \). The profits are divided evenly among the 100 travelers.

a. What is aggregate demand for Art onboard V-Forge? State it as precisely as possible.
b. Completely determine and summarize the competitive equilibrium.
c. What is the production possibility frontier for final goods, focusing on the total amount of Art and the number of servant Hueies? Does the competitive equilibrium lie on the frontier?
9. Suppose a two-person, two-good economy where

\[ U_a = \min \left( \frac{X_a + Y_a}{2} \right) \quad \text{and} \quad U_b = X_b^{1/2} Y_b. \]

Total X in the economy is 9 units.
Total Y in the economy is 18 units.

a. Find an equation for the contract curve and justify your answer.
b. Find an equation describing the utility possibility frontier.
c. Suppose "A" owns all the Y (18 units) and "B" owns all the X (9 units). Find the competitive equilibrium.
d. For the following social welfare function find the optimal allocation of X and Y:

\[ SW = U_a U_b^{2/3}. \]

10. Consider an economy comprised of M identical consumers and N identical firms producing a single consumption good x. Technology is described by the function, \( x = L^{1/2} \); consumer preferences over leisure (H) and consumption are represented by the utility function, \( U(x, H) = xH \), where \( L + H \leq 24 \) for each individual.

a. Suppose each consumer shares equally in the ownership and profits of each of the N firms. Determine the competitive equilibrium for this economy. Then determine how the welfare of a consumer will vary with the number of firms and consumers in this economy.
b. Set up a Lagrangian representing the Pareto problem for this economy using 4 decision variables \((x_n, L_n, x_m, H_m)\) in addition to the necessary Lagrange multipliers. Obtain first-order conditions for each of these 4 types of decision variables. Process these 4 conditions algebraically until all Lagrange multipliers have been eliminated. [How many points are described by the result?] Check to see if the competitive equilibrium computed in part (a) satisfies this condition.

11. In a two-person economy containing three goods \((x, y, z)\), individuals A and B have utility functions given by

\[ U_a = x_a y_a^2 z_a \quad \text{and} \quad U_b = x_b y_b^3 z_b^2 \]

Initial endowments are

\[ I_a = (220, 4, 4) \quad \text{and} \quad I_b = (220, 2, 3). \]

Fully demonstrate the Pareto optimality of the competitive equilibrium that you obtain for this situation. In your work please denote prices as 1, p, and q, respectively, for x, y, and z.

12. Given any Bergsonian welfare function \( SW = f(U_a, U_b) \) and specific utility functions for each individual who is included in the welfare function, can one always determine an equivalent social welfare function which has total amounts of each good as dependent variables, i.e., \( SW = g(X, Y) \)? Experiment with this using selected functional forms.

13. For a two-output economy utilizing only labor and capital, graph the contract curve in input space and describe the production possibility frontier mathematically.

\[ X = L_x^{1/3} K_x^{2/3} \quad \text{and} \quad Y = 4L_y + K_y. \]
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Total available L is 5 units.
Total available K is 20 units.

Does the production possibilities frontier have the proper slope everywhere? What is the MRT along this frontier?

14. A miner and a woodsman each have the potential to use local government resources for their respective production activities. The miner extracts ore, R, and the woodsman cuts timber, T, with each producer wishing to use the government's land, L, and water, W. The government owns the only available L and W, so markets for these inputs are nonexistent. Production functions are

\[ R = 4L^{1/4}W^{1/2} \]  
\[ T = 3L^{1/3}W^{2/3} \]

Available government resources to be allocated between the miner and the woodsman are \( L = 80 \) and \( W = 10 \). Product prices are determined exogenously by national markets: \( p_R = 2 \) and \( p_T = 1 \).

As the local official having total responsibility for managing these resources, you can employ any means that you can logically justify for the allocation of L and W.

a. Use Pareto optimality to guide your decision.
   - Find the production possibilities frontier.
   - How many Pareto optimal states are there?
   - Select and precisely specify (quantitatively) a preferred policy for achieving Pareto optimality in this setting. What economic state will result from this policy?

b. Use potential Pareto optimality.
   - Maximize the total value of ore and timber production. [Why does this give the potential Pareto optima?]
   - How many potential Pareto optimal states are there?
   - Select and precisely specify (quantitatively) a preferred policy for achieving potential Pareto optimality in this setting. What economic state will result from this policy?

c. Discuss the distinctions you've found between these two norms for economic efficiency.

Market Failure

15. Consider a society of two individuals (Jones and Kay) with two goods (X and Y). Let \( X_i \) and \( Y_i \) \((i = J, K)\) denote individual i's consumption of X and Y. The preferences of J and K are known to be represented by

\[ U_J = X_J Y_J \]  
\[ U_K = X_K^2 Y_K / Y_J \]

Initial endowments are:

Jones has 8 X and 8 Y;  
Kay has 10 X and 16 Y.

a. Determine the competitive equilibrium and the resulting utility levels.

b. As you can see from Kay's utility function, he suffers from a "keep up with the Joneses" complex regarding the consumption of Y. Does this particular externality result in a competitive equilibrium which is not Pareto optimal? Analytically verify your answer by demonstrating Pareto optimality (or nonoptimality) or by providing a counterexample.

16. Individuals A and B both derive utility from the consumption of Food and all other Goods. A is more wealthy, but she is also altruistic because she derives utility from B's consumption of food. Utility functions are
The initial endowment is \( \{ F_A, G_A, F_B, G_B \} = (10, 10, 2, 2) \).

Show whether or not self-interested market behavior will lead to economic efficiency in this situation.

17. Suppose that Texas produces only two goods, beef (B) and electricity (E). When a high-sulfur fuel such as lignite is used to generate electricity, sulfur oxides are emitted as air pollutants which then react with atmospheric moisture to form a weak sulfuric acid. Acid rain then falls and damages the forage which cattle feed upon. Beef is produced in Texas from costless forage (F) according to the following production function:

\[ B = \frac{F^2}{2} \]

Assume a short-run situation in which plant capacity limits electricity generation to 30 units. Texas electricity is produced from petroleum (P) and lignite (L) according to the following production function:

\[ E = 3P + 2L \]

The acid rain relationship is given by:

\[ F = 400(1 + L)^{1/2} \]

Prices are exogenously determined and are as follows:

\[ P_P = 10, \ P_F = 0, \ P_E = 15, \ P_B = 40, \ \text{and} \ P_L = 22. \]

Suppose that beef production and electricity production are managed by two different profit-maximizing individuals. What is the competitive equilibrium? Are the state's resources being allocated efficiently in this competitive equilibrium?

18. Two neighboring irrigators use their wells to pump groundwater from the same aquifer. Due to the water table drawdown caused by operating the wells, each irrigator's cost of groundwater withdrawal is partially dependent on pumping by the neighbor. Pumping costs for irrigators 1 and 2 are given by the following functions where \( x_i \) is the amount of groundwater pumped by irrigator i:

\[ C_1 = 4 + 2x_1 + \frac{1}{4}x_1^2 + d_1x_2 \quad \text{with} \quad 0 < d_1 < 4 \]

\[ C_2 = 4x_2 + \frac{1}{4}x_2^2 + d_2x_1 \quad \text{with} \quad 0 < d_2 < 1.3 \]

The factors \( d_i \) represent the degree of interaction between the two irrigators' wells. These factors vary (independently) within the specified ranges from year to year, but their values are known at the beginning of each cropping season. Production functions are identical and are given by \( y_i = 16x_i - \frac{1}{2}x_i^2 \) and the price of y is \( \frac{1}{2} \).

a. Assuming no cooperation between farms, find the profit-maximizing levels of groundwater withdrawal and resulting profit for each farm.

b. Is the result of part (a) economically efficient? Explain and show.

c. How do you set regulations to address this problem? Does your policy improve each farm's profit? Does your policy improve economic efficiency?

19. A region's 45 identical and independently owned farms use pesticides and many other inputs to grow cotton. Because of the economic activity and employment that these farms support, the regional government prefers that this group of farms be as collectively profitable as possible. Each farm's application of pesticides is a benefit for other farmers because the destroyed pests cannot reproduce or invade neighboring farms. Once profit-maximizing choices for all nonpesticide inputs are substituted into each farm's production function, the production function becomes
\[ c_i = 15 \left( x_i + \frac{1}{100} \sum_{j \neq i}^{45} x_j \right)^{1/2} \forall i. \]

where
\[ c = \text{amount of cotton produced by a farm}, \]
\[ i, j \in \{1,2,3,...,45\}, \text{ and} \]
\[ x = \text{amount of pesticides employed by a farm}. \]

The price of cotton is $4 per unit. The price of x is $1 per unit, and all other employed inputs have a total cost of $1000 per farm.

a. What type of social problem is present in this scenario? What social policies might be attractive? Select a preferred policy type and discuss why you think it is the best one.
b. Quantitatively analyze the pesticide decisions of this region's farms, and quantitatively show the problems that arise when no policy is used.

20. A flat planet's north side is occupied by a united group of people called Mopeds, and the other side is occupied by a similarly united group called the Bipeds. The welfares of both groups depend on their own consumption of goods (G) and environment (E). Both Mopeds (M) and Bipeds (B) obtain goods by applying technology to the environment. Using subscripts to denote group, each region's technology is given as follows:
\[ G_B = 4 - E_B \quad \text{and} \quad G_M^2 = 16 - E_M^2. \]

Each of these two technologies is fixed and constant. An issue with both technologies is that they spillover in the sense that production has a negative influence on the other side of the world. The Mopeds possess a superior technology, but it also causes a greater spillover. This situation is captured by the two following welfare functions.
\[ W_B = \frac{G_B^2 E_B^2}{G_B^2} \quad \text{and} \quad W_M = \frac{G_M^3 E_M}{G_B}. \]

These two groups cannot engage in any trade, because they would fall off the world.

a. Fully characterize the choices and consequences of independent decision making for the Mopeds and Bipeds.
b. Demonstrate the economic efficiency or inefficiency of independent decision making by the two groups.

21. Assume an economy consisting of two individuals, A and B. The production of electricity (E) requires lignite (N) and labor (L) according to the following production function: \( E = N^{1/2} L^{1/4} \). Individual A owns the only power plant (and is, therefore, entitled to all profits), and this person supplies no labor. Individual B supplies 100 units of labor regardless of the wage rate. One hundred tons of lignite is available and this resource is jointly and equally owned by A and B. Preferences can be represented by the following utility functions:
\[ U_a = E_a^2 \quad \text{and} \quad U_b = E_b^{0.1} Q^{0.1} \]

where Q is an index of environmental quality. Environmental quality is negatively related to electricity production but can be improved by applying some electricity to environmental clean-up. The "production
function" for Q is as follows: \( Q = \frac{E_q}{E} \). Clearly, \( E_a + E_b + E_q = E \). As price variables use \( p_e, p_n, \) and 1.

[That is, labor is the numeraire.]

a. Assume that B does not purchase any electricity for environmental improvement. Therefore, \( U_b = 0 \).
   i. Find supply and input demand functions for the firm.
   ii. Find the competitive equilibrium.
   iii. Determine \( U_a \) and \( U_b \).

b. Determine the Pareto optimal relationships among \( E_a, E_b, \) and \( E_q \). [Hint: The Langrangian can be stated so that two of these three variables are the only choice variables.]
   i. Is the answer to a(ii) above Pareto optimal?

c. Case I: B must bear the burden of the externality.
   i. Solve the competitive equilibrium problem with the following modification. Substitute \( \frac{E_q}{E} \) for \( Q \) in B's utility function in order to recognize B's opportunities for purchasing electricity for environmental improvement. Note that B must regard E as an exogenously determined quantity.
   ii. Is your answer to c(i) Pareto optimal?
   iii. What is \( U_a \) and \( U_b \)?

d. Case II: A must bear the burden of the externality.
   i. Suppose that B has a property right to perfect environmental quality \( (Q=\infty) \). This would imply that \( E = 0 \). Let \( t \) be the price charged by B for each unit of E produced. Any \( E_q \) must still be paid for by B. Find the competitive equilibrium. [Hint: Each unit of electricity production now has a value of \( p_e - t \) to the firm. Define \( p^* = p_e - t \) and restate and resolve the firm's profit maximization problem where \( p^*, p_n, \) and 1 are the only prices. Restate and resolve B's problem where \( E_b, E_q, \) and E are now decision variables. E is a decision variable because B must decide how much of the "E property right" to supply as a function of \( t \). Knowledge of \( p^* \) from the firm's problem and B's first-order conditions will enable you to determine this competitive equilibrium.]
   ii. Is your answer to d(i) Pareto optimal?
   iii. What is \( U_a \) and \( U_b \)?

e. How do your answers in a(iii), c(iii), and d(iii) compare?

f. Review and discuss your findings in b(ii), c(ii), and d(ii) in relation to externality theory.

22. A local 2-person economy operates in a broader market-oriented economy, and neither person has the power to influence prices established in the broader economy. These prices therefore stand as both private and social values for the local economy.

Guy A derives utility directly from the profit obtained from his factory \( (U=\pi) \). He produces \( y \) for export according to the production function \( y = m^{1/3} n^{1/3} \). \( m \) and \( n \) are imported inputs. Prices for \( y, m, \) and \( n \) are given by 8, 4, and 2.

Gal B derives utility from the consumption of three goods: leisure \( (e) \), food \( (f) \), and quiet \( (q) \). Her preferences are given by \( U = efq \). She is endowed with 40 units of leisure. Currently, she can export all or part of her leisure at a price of \$1 per unit and use the income to import food at \$2 per unit.

If A's factory was idle, B would enjoy a level of quiet equal to 1. Otherwise, the level of quiet is given by \( q = (m+1)^{-1} \).

As you should know and may assume for the remainder of this problem, a Pareto optimal policy for this local externality is to endow A with the right to use \( M \) units of m and allow him to contract with B for
expansions or reductions in m use. This policy obligates B to endure \( m = \bar{M} \), but no more than that quantity unless she is paid. She may also buy all or part of \( \bar{M} \) from A.

a. Let \( t \) denote the per unit price of M rights. Find supply and demand functions for M rights assuming rational behavior by both A and B.

b. Which version of the Coase Theorem is (or should be) supported by your findings in part (a)? State this version completely, and clearly indicate why it is supported by the results of (a). [No math needed for this part.]

23. The local supply of deer (for hunting) is controlled by ranchers 1 and 2 who own adjacent property. These ranchers spend money on management, M, to produce deer, D. The ratio of the price of D to the price of M is dictated by regional markets and is given by \( \frac{P_D}{P_M} = 4 \). Because of terrain, vegetative cover, etc., some deer migrate from ranch 2 to ranch 1 during the hunting season. [Fences to retain cattle are ineffective for deer.] An interdisciplinary research team composed of range scientists and an economist (you) believe that annual production functions are given by

\[
D_1 = \left( M_1 + \frac{1}{2} M_2 \right)^{1/2} \quad \text{and} \quad D_2 = M_2^{1/4}
\]

a. Determine whether or not independent decision making will result in economic efficiency. What is happening here? Can independent decision making be efficient if transaction costs are nonzero?

b. If rancher 2 erects a deer-proof fence, we believe the production function will be altered to \( D_2 = M_2^{1/2} \) because migration is eliminated. [The effect on ranch 1's production function is obvious: set \( \frac{1}{2} M_2 = 0 \).] Under these conditions what is the rancher's maximum willingness to pay (annually) for the fence? Assuming that the fence is profitable, will the rancher's decision to erect the fence be economically efficient?

24. As the economist on the team studying the above question, you possess very critical responsibilities beyond answering parts a and b above. Specifically, you are charged with designing and analyzing alternative policy measures. Your policy recommendations will greatly influence the decision-making process, so you want to consider and compare all policies which may be promising.

Identify and discuss the policy alternatives which may be appropriate for this setting. Try very hard to resolve a preferred policy. Separate your response into two cases.

Case A: the narrow situation depicted in the previous problem including the possibility for fence building.

Case B: an expanded situation in which the previous problem only lends insight. Here, there are numerous ranches of varying character throughout the region. Production functions vary and are difficult to determine.

25. The university has 80 laboratories (L) and 80 doctorates (D) to use to produce education (E) and research (R) according to the following production functions.

\[
E = L_0.5 D_0.5 + kR \\
R = \ln \left( \frac{L_4}{R} \right)
\]

where \( k \) is exogenous and unspecified at this time.

The university's president wishes to be economically efficient in the allocation of university resources.

a. What combinations of E and R are efficient for the university to produce?
b. Briefly discuss your interpretation of the above scenario.

c. In what way does your answer to (a) change as k increases? Why?

d. If E and R were purely rival goods so that markets could provide these goods, could PO be achieved? [No math please for part d]

26. Suppose that there exists a pollutant that is known to be emitted by a certain industry, but it is impractical to observe emissions from individual firms. Suppose that it is, however, possible to observe firms’ use of pollution-causing inputs. Then it becomes possible to attach policy to input use instead of emissions. Use this opportunity in the following setting.

Firms 1 and 2 in the same pollutionshed produce good y using inputs n and w. Market-determined prices for y, n, and w are 20, 2, and 2, respectively. Production functions are given by

\[ y_1 = 2n_1^{0.5}w_1^{0.25} \text{ and } y_2 = n_2^{0.5}w_2^{0.25}. \]

These two firms are known to be emitting a pollutant called z. Their individual pollution contributions are not exactly known but they are estimated to be related to input use according to these linear relationships:

\[ z_1 = n_1 + w_1 \text{ and } z_2 = 2n_2 + 2w_2. \]

Pollution damages are given by \( 0.25(z_1 + z_2) \).

Your tasks are to completely specify two efficient policies for this setting – one is to be an efficient Pigouvian tax policy (for the inputs) and the other is to be efficient command-and-control regulations (for the inputs). You may presume that the policy objective is to achieve maximum net social benefits (i.e. potential Pareto optimality). To avoid confusing the managers of these firms, you must state each policy very clearly. Which policy do you recommend?

Another version of this question is to assume that the damage function is unknown. Suppose that the goal is then to achieve a pollution standard with maximum net social benefits. The constraint is \( z_1 + z_2 \leq 5200 \).

Again, Pigouvian and regulatory policies are available. The algebra here is cumbersome, and numerical methods may be preferable.

27. An annually renewed resource pool is used by n+1 identical individuals. The net returns of resource extraction for each of these individuals depend on summed extraction by all other users as well as on own extraction. The net return function of every user is the same:

\[ NR_i = a + bf_i - cf_i^2 - d \sum_{j \neq i} f_j \]

where a, b, c, and d are known, positive parameters and \( f_i \) is own resource extraction. It is also known that \( b > 2dn \).

a. What are total net benefits of this resource pool when individuals ignore their interdependence and act independently?

b. What can the pool's net benefits be if individuals act cooperatively? What is the total improvement in net benefits over independent behavior (part a)?

c. Quantitatively establish two distinct policies that the government can use to achieve the results of part b.

28. The world's many environmental and natural resource issues are firmly linked to global population policy. Explore this issue using the following model components. Every individual's lifetime utility function and budget constraint can be given by

\[ U_i = f_i(x_i; k; K) \text{ and } x_i + pk_i \leq M_i \]
where \( x_i \) is the individual's consumption of a composite good,
\( k_i \) is the number of children the individual has,
\( K \) is the total children of all individuals,
p is the personal cost of having a child, and
\( M_i \) is income (in units of the composite good).

\( x_i \) and \( k_i \) are the individual's only decision variables. Logically, we have

\[
K = \sum k_i.
\]

The marginal utility of \( x_i \) and \( k_i \) are nonnegative, but the marginal utility of \( K \) is nonpositive because the impact of higher \( K \) is to accelerate environmental degradation and resource depletion. Your investigation should employ economic efficiency as the goal of population policy and should suggest and discuss some preferred policy(ies). You should augment the model with whatever discussion or additional details you feel are pertinent.

29. It has been suggested that industrial activities pave the way for municipal and industrial growth through the negative influence of industrial air pollution upon nearby agricultural yields and profits and, therefore, agricultural land values. The chief agents in this process are ozone and sulfur dioxide which defoliate or injure crops. These gases are byproducts of fuel combustion by industry.

Your task is to examine and recommend policy for a hypothetical local scenario given as follows. The agricultural sector is centrally managed by a single cooperative, and the cooperative's net annual profits are

\[
\pi_t^A = 10 - P_t
\]

where \( t \) is an index of years \( (t=0, 1, 2, \ldots) \) and \( P_t \) is air pollution in year \( t \). Pollution is a function of total industrial fuel consumption in \( t \):

\[
P_t = F_t^{1/4}
\]

All industrial plants have an identical annual profit equation:

\[
\pi_i = 8f_i - f_i^2
\]

where \( f_i \) is the plant's fuel consumption. [Note: Prices are imbedded in the above profit functions and are assumed constant.] There are no plants in year 0, and this number increases by 1 every year without fail. Therefore,

\[
N_0 = 0
\]

and

\[
N_t = N_{t-1} + 1 = t.
\]

Finally, aggregate fuel consumption and industrial profit are obviously given by

\[
F_t = N_t f_t \quad \text{and} \quad \pi_t = N_t \pi_t.
\]

a. Assuming no policy is adopted, determine \( \pi_t^A \) and \( \pi_t \) as a function of \( t \) only (assuming profit maximization in industry). Which year is the first when the agricultural cooperative decides to produce nothing?

b. Devote careful thought to devising and completely specifying a policy which is both efficient and equitable. Discuss your selection by explaining the merits of your proposal and the reasons that it is preferred over other specific policies. Does your policy have any disadvantages? Are there other policies with similar characteristics to your selection?
30. Two nearby firms produce the same output, $y$. Each benefits from the other's use of the only input according to the following production functions:

$$y_1 = 2x_1^{1/2}x_2^{1/4} \quad \text{and} \quad y_2 = x_1^{1/2}x_2^{1/4}$$

where the subscripts refer to the firm. Thus, for example, firm 1 controls $x_1$ but not $x_2$. Assume that the price of $x$ is 1 and that the $y$ industry is composed of only these two firms.

a. Discuss this problem setting in relation to natural resource theory. Illustrate this setting generally using two concepts of the industry's supply curve for $y$, PMC and SMC (private marginal costs and social marginal costs). Use these graphs to show the welfare impact associated with independent action by these two firms as opposed to collaboration.

b. Compute equations for the graphs used in part a, and use these equations to calculate the welfare effects illustrated in part a. You may select any reasonable function you wish for $y$ demand.

31. Expand the general economic model developed in this course in order to investigate Pareto optimality and competitive equilibrium for a public good. Accomplish this using the following procedure.

a. Let $z$ be the public good (its publicness applies to consumption only). Let $p_z$ be its equilibrium price. Find the two general conditions which represent profit maximization by firms (eqs. la and lb).

b. Let $z^k$ be the amount of $z$ purchased by individual $k$. Every individual consumes an amount $z$, and it seems reasonable to presume that

$$\frac{\partial z^k}{\partial z} \equiv 0 \quad \text{for all } k.$$

Find the first-order conditions for utility maximization (eqs. 2a and 2b).

c. Set up the Pareto problem and generate equations (3a) – (4b).

d. Do competitive markets yield Pareto optimality? Explain how your results demonstrate this. (Equations 1 and 2 from the Kuhn-Tucker part of a handout may assist you in understanding 2b).

e. Use the results of (c) to obtain Samuelson's condition for Pareto optimality with a public good, i.e.,

$$\sum_k MRS = MRT.$$

32. Individuals 1 and 2 consume goods $X$ and $Z$. Their preferences are as follows:

$$U^1(X_1,Z) = 2X_1 + Z^2 \quad \quad U^2(X_2,Z) = 2X_2 + 2Z^2.$$ 

This two-person economy possesses production technologies which yield the following production possibility frontier:

$$X^2 + Z^2 = 25 \quad \quad X,Z \geq 0 .$$

a. Suppose that $X_1=2$, $X_2=2$, and $Z=3$. Determine all Pareto optima to see if this situation is efficient.

b. If your answer to the first question is "yes," then use the following social welfare function to maximize social welfare: $SW = U^1U^2$.

c. If your answer to the first question is "no," then determine a Pareto superior reallocation from the initial situation such that production efficiency is maintained.

d. Select a numeraire good and determine an efficient Lindahl equilibrium for this economy under the assumption that individual 1 is entitled to one-third of the economy's revenue.
33. In a two-person society, individual A's demand for mosquito control is \( Q_a = 100 - p \), and B's demand is \( Q_b = 200 - p \). Assume that mosquito control is a pure public good that can be produced at a constant marginal cost of $120.

   a. What is the optimal level of mosquito control?
   
   b. If mosquito control were left to the private market, how much might be produced? Is the private amount optimal?
   
   c. If the government were to produce the optimal amount of mosquito control, how much would the total costs be?
   
   d. How should the tax bill be allocated between the two individuals if they are to share it in proportion to the benefits each receives from mosquito control?
   
   e. What Lindahl prices are consistent with motivating efficient action?

34. Communities A and B lie on opposite sides of a large lake. Both gain social utility from material wealth, \( M \), and suffer social disutility from water quality degradation, \( D \). Technology causes \( M \) and \( D \) to be positively related in each community because of water pollution caused by \( M \) production. The following production possibility frontiers are relevant:

   \[
   D_A = 2M_A^2 + \frac{1}{2} \quad \text{and} \quad D_B = 2M_B^2 + 2.
   \]

   Both communities are injured by their own pollution discharges as well as by the discharge of each other. Because community A lies "uplake" from B, A is not fully affected by B's discharge. Social welfare functions are as follows:

   \[
   w_A = \ln(M_A) - \ln \left( D_A + \frac{1}{2}D_B \right) \quad \text{and} \quad w_B = \ln(M_B) - \ln \left( D_A + D_B \right)
   \]

   a. What type of market failure would you say is present? Explain.
   
   b. Demonstrate the economic inefficiency of independent decision making by communities. Define what you mean by economic efficiency within your response.
   
   c. Describe (qualitatively) a particular policy measure for alleviating this problem. What things would be altered by this policy? Can you specify parameters for the policy you are describing?

35. In a many-consumer economy possessing a bridge, each consumer's utility is given by

   \[
   U_i = A_i^3 \left( \frac{B_i}{K_i} \right)
   \]

   where \( A_i \) is individual i's consumption of "all other goods," \( B_i \) is the number of bridge crossings by i, and \( K_i \) is a known constant. Each consumer begins with an endowment, \( E_i \), of "all other goods." There is no debt or costs associated with the bridge. Because the bridge is a nonrival good, bridge crossings are unlimited in supply. Costless exclusion is feasible.

   a. Construct the Pareto problem for this economy where all \( A_i \) and \( B_i \) are decision variables in your construction. Derive a set of first order conditions depicting Pareto optimality.
   
   b. Determine the Marshallian demand for bridge crossings by the arbitrary individual. How should bridge crossings be priced and why?
   
   c. Improved research has determined that \( K_i = K_i \sum_{j \neq i} B_j \).
That is, consumers are sensitive to bridge congestion. \( k_i \) is individual \( i \)'s crowding sensitivity coefficient and \( \sum_{j \neq i} B_{ij} \) is total bridge crossings by all consumers other than one's self. How do your conditions for economic efficiency and your optimal pricing strategy change as a result of this new information?

36. Two identical and neighboring communities each consume only two goods, \( Y \) and \( Z \). Their respective community welfare functions are

\[ W_a = Y_a + Z \quad \text{and} \quad W_b = Y_b + Z. \]

where \( Z = Z_a + Z_b \). These two communities also have identical productive transformation opportunities between these goods:

\[ Z_a^2 = 4 - 2Y_a \quad \text{and} \quad Z_b^2 = 4 - 2Y_b. \]

a. Because these communities are identical in every respect, there are no comparative advantages, and it will not be advantageous to engage in any trade. For each community, find its socially optimal level of \( Y \) and \( Z \) production. What is the resulting index of welfare for each community?

b. Set up a single Pareto-like problem in which you maximize \( W_a \) subject to a fixed and arbitrary level of \( W_b \) and the technological constraints. Substitute the results of (a) into your FOCs to determine whether these results are Pareto optimal.

c. Why should you expect the final result obtained in (b)?

37. On a two-nation planet, each country obtains welfare from the goods (\( G \)) it produces, and it also obtains welfare from the quality of the planet's atmosphere. Greater atmospheric quality can only be achieved by sacrificing some goods in return for abatement (\( A \)). The technological tradeoff between goods and abatement for each country is given by

\[ G_1 + \frac{1}{2}A_1 = 100 \quad \text{and} \quad G_2 + \frac{1}{4}A_2 = 50. \]

Nation-specific welfare functions are given by

\[ W_1 = G_1^2(A_1 + A_2) \quad \text{and} \quad W_2 = G_2^2(A_1 + A_2). \]

a. What production bundles do the two nations produce when they engage in independent action?

b. Demonstrate that the planet's people are not well served by the economic state identified in part (a).

c. (No numbers necessary here.) Discuss this issue and recommend a single, clear policy measure. Indicate the nature of the improvement offered by your policy.

Alternative question (instead of a): Plot each nation's private provision (Nash) reaction curve (one for each nation) in \( A_1-A_2 \) space. What does the intersection of these curves represent?

Alternative question (instead of b): Now, set up the appropriate Pareto problem for this economy where you utilize the technology constraints to eliminate variables \( G_1 \) and \( G_2 \) from the problem. Processing the first-order conditions for \( A_1 \) and \( A_2 \) produces a somewhat complicated quadratic relation between \( A_1 \) and \( A_2 \). Use of the quadratic formula allows you find the Pareto frontier (expressing \( A_2 \) as a function of \( A_1 \)) and to see the relation between the reaction curves and the Pareto frontier.

38. Drivers 1 and 2 derive utility from driving according to utility functions

\[ U_1 = U_1(R,G_1) \quad \text{and} \quad U_2 = U_2(R,G_2) \]

where \( R = \) miles of roadway and
\[ G_i = \text{gallons of gas consumption by i.} \]

The production possibility frontier for this two driver economy is given by

\[ H(R,G) = 0 \]

where \( G_1 + G_2 = G \).

a. Derive a complete set of efficiency conditions for allocating all goods in this economy. [Simplification of your conditions is not required].

b. Now suppose

\[ U_1 = RG_1, U_2 = R^2G_2 \text{ and } H(R,G) = R^2 + G^2 - 16. \]

give a particular example of efficient consumption and production bundles for this economy.

c. How might a market system be modified to drive this economy to the efficient state you identified in part b? [Merely discuss. Numbers are unnecessary.]

39. The management of the local TV station feels that there are only two types of viewers (\( \alpha \) types and \( \beta \) types) within its signal range. Let \( T \) be an index of television output, and let \( B \) denote a homogeneous bundle of all other goods. Let \( P_B = 1 \). Utility functions for individuals of the two types of viewers are

\[ U_{i\alpha} = TB_i^4 \text{ and } U_{i\beta} = TB_j^9 \]

There are 200 \( \alpha \) viewers with individual incomes of 20 and 200 \( \beta \) viewers with individual incomes of 10. The station's supply function is given by \( P_T = 10T \), and the station can exclude costlessly.

a. What type of good are we dealing with? What must the station do in order to achieve economic efficiency in the production and allocation of its TV signal? Why? Discuss.

b. Determine the equilibrium corresponding to your recommendations in part (a).

c. Instead of operating as a private venture (ignore a and b), suppose the station is publicly owned and finances its operations using a lump-sum tax. The station is providing TV at the level of \( T = 4 \), and net income levels are \( I^{\alpha} = 19.8 \) and \( I^{\beta} = 9.8 \). All net income can be spent on \( B \). It has been proposed that TV production be raised to \( T = 8 \), in which case net income levels will be \( I^{\alpha} = 19.2 \) and \( I^{\beta} = 9.2 \). For an \( \alpha \) individual, calculate a COMPLETE Hicksian measure of welfare change for this proposal, presuming you will be interested in employing the Kaldor test.

40. Consider a two-person economy in which both individuals consume only leisure (\( H \)) and public good (\( G \)). These individuals have the following utility functions:

\[ U_a = H_a^2G \text{ and } U_b = H_bG. \]

Each individual has equal stake in the only firm capable of producing \( G \). During any given week, labor (\( L \)) is used in the production of \( G \) according to the following production function:

\[ G = 4L \leq 0. \]

Each individual has 60 hours per week; this time can be devoted to either labor or leisure.

a. Find the Lindahl equilibrium.

b. Demonstrate that the Lindahl equilibrium either does or does not satisfy the Samuelsonian condition for the Pareto optimal provision of public good.
41. In a 2-output economy, material (M) is produced using land (L) and capital (K). The material-producing sector has the aggregate production function

\[ M = 24L^{0.5}K^{0.25}. \]

The 2nd output, atmospheric quality (Q), is produced as a consequence of idle land according to

\[ Q = L. \]

Overall, the economy's resources are \( L=80 \) and \( K=16 \). All four commodities are traded in competitive markets, and everything is privately owned to the extent possible. In market equilibrium Q is 3 times more valuable than M. However, analysts consider Q to be a nonrival and nonexcludable good, and they suspect that its true social value is 4 times greater than the social value of M.

a. Find the economy's production possibility frontier.
b. Recommend, justify (relative to other possible policies), and quantify a specific policy measure.
c. Perform an economic analysis fully quantifying the value of your recommended policy.

**Single Agent Welfare Analysis**

42. Choose two PO points lying on a contract curve that results from an earlier problem. Do not select the end points. Designate one as the initial (prepolicy) point and one as the subsequent point. Compute CV for each person and then add the two (or more) CV’s. Is the total positive or negative? What are your general conclusions regarding the power and normative implications of alternative definitions of efficiency?

43. Let there be a single firm with the profit function \( \pi(p,w_1,w_2) = \frac{4p^4}{w_1w_2^2} \).

a. Use this function to obtain appropriate supply and demand functions representing this firm's profit-maximizing behavior for the firm's product, q, and its two inputs, \( x_1 \) and \( x_2 \).
b. Initial, prepolicy prices are (100, 6, 12). Subsequent postpolicy prices are (100, 9, 12). In three separate calculations, use the results of part (a) to calculate a correct welfare measure for this policy scenario. [You may not use the profit function beyond its use in part (a).] Provide graphical depictions for each of these three calculations.

44. Change at least one the exponents in the prior question's profit function. If you wish, you can make other changes to the function as well. Do not alter any of the notation.

a. Use this function to obtain appropriate supply and demand functions representing your firm's profit-maximizing behavior for product, q, and the two inputs, \( x_1 \) and \( x_2 \).
b. Clearly state and distinguish initial and subsequent price vectors which are different than the ones used in the prior problem. Have only the \( w_1 \) input price changing while the other two prices are unaffected by the policy. In three separate calculations, use the results of part (a) to calculate a correct welfare measure for this policy scenario. [You may not apply the profit function beyond its use in part (a).] Provide graphical depictions for each of these three calculations.

45. Given the production function \( y = 10x_1^{1/4}x_2^{1/4} \):

a. Find the cost-minimizing conditional input demand functions, \( x_1(w_1,w_2,y) \) and \( x_2(w_1,w_2,y) \), as well as the associated cost function \( C = C(w_1,w_2,y) \).
b. Using profit \( \pi = py - C(w_1,w_2,y) \) and a part (a) result, differentiate \( \pi \) once to derive the supply function \( y = y(w_1,w_2,p) \).
c. Using the supply function and the conditional input demand functions derived above, obtain the input demand functions for profit maximization.

d. Using the input demands (part c) and supply (part b) with \( w_2 = 4 \) and \( p = \frac{4}{5} \), calculate the change in quasi-rent when \( (w_1) \) is changed from 9 to 16. Demonstrate that the welfare change is the same in both the input \((x_1)\) and output markets, and include graphical sketches of the required demand and supply curves as well as the area pertaining to the change in quasi-rent.

e. For \( w_1=16 \) and \( w_2=4 \), calculate the change in quasi-rent due to an output price change from \( \frac{4}{5} \) to \( \frac{8}{5} \) units. Show that this change is the same for both the input \((x_1)\) and \((x_2)\) and output markets. Illustrate graphically.

f. With \( w_2 = 4 \), calculate the change in quasi-rent using any single market \((x_1, x_2, \text{or } y)\) where \( w_1 = 9 \) and \( p = \frac{4}{5} \) initially, and \( w_1=16 \) and \( p = \frac{8}{5} \) subsequently.

46. Using only wood, \( w \), and chlorine, \( c \), a segment of the pulp and paper industry experiences the following production function in aggregate:

\[
y = w^{1/3}c^{1/3}
\]

where \( y \) denotes the quantity of paper produced.

a. Derive this group's supply and input demand functions.

b. Suppose the initial prices of paper, wood, and chlorine are $6, $4, and $5. Calculate the changes in quasi-rent when the price of paper changes to $3. Is the quasi-rent change as measured in all three markets the same? Show the answer by computing all three.

c. Suppose the price of wood changes from $4 to $2 as the price of paper changes from $6 to $3. What is the change in quasi-rent for this combined price change? Show whether this setting exhibits path dependence.

47. A certain firm producing good \( y \) with inputs \( x_1 \) and \( x_2 \) operates with the following cost function:

\[
C = y^2\left(\frac{w_1}{w_2} + \sqrt{w_1w_2} + w_2\right) + F
\]

where \( F \) denotes fixed costs.

a. Calculate the firm's supply function, \( y(p, w_1, w_2) \) presuming firm actions do not influence prices, as is customary.

b. Suppose \( F \) is equal to 0. Prices are initially \((p, w_1, w_2) = (36, 12, 12)\), and these subsequently change to \((36, 9, 16)\). Accurately illustrate in \((y, p)\) space the before and after supply curves and the quasi-rent change resulting from these price changes. Calculate \( \Delta R \) in two ways:

i. Calculate the area as illustrated in your graph.

ii. Calculate the difference in before and after profit using \( \pi = py - C \).

c. Repeat all of part b using \( F = 8.88 \).

48. For an imaginary firm using inputs and producing outputs of your choice, construct an appropriate set of data containing at least 5 observations which do not produce a perfect statistical fit. The number of inputs/outputs is up to you. Use this dataset as the basis for the following tasks.

a. Statistically estimate the firm's production function by OLS. Whether or not parameter estimates are statistically significant is not a concern. The production function should, however, be "well behaved".
Derive supply and/or demand functions (as needed) using this production function. Select initial price levels for all inputs and outputs. Suppose that a particular input or output price is changing by some determined amount (you select the amount). Perform the integration necessary to find ΔR for this price change. Illustrate this latter task with a graph.

b. For the same situation and price change examined in part (a), find ΔR by integration in a different market than you used in part (a). Also illustrate.

Only part (a) is required. Part (b) is 20% extra credit and is optional. Turn in: (page 1) your dataset with inputs/outputs clearly labeled; (page 2) your statistical computations or summary output (a copy of canned program output is okay); and (remaining pages) the computations needed to complete the assignment. Only 2 pages are allowed for the initial material. Clearly define initial and subsequent price vectors.

49. Suppose a given restricted profit function, \( \pi(p, w_1, w_2, k) \), for a firm using only three inputs where \( p \) is output price, \( w_1 \) and \( w_2 \) are input prices, and \( k \) is a fixed input. The following information is given:

\[
p^0, w_1^0, w_2^0; p^1, w_1^1, w_2^0, \text{ and } k.
\]

Suppose: \( \pi^0 = \pi(p^0, w_1^0, w_2^0, k) < 0. \)

Is the change in quasi-rent \( \pi^1 - \pi^0 \), \( -\pi^0 \), or something else? Why?

50. Suppose an output price change of \( m \) units is predicted to occur for a firm over a period of three years: 30% in year one, 50% in year two, and 20% in year three. The production function for this firm is completely known to you. Some of the inputs are durable, and their productivity is a function of age. None of the durable inputs are productive after reaching two years of age. How would you conduct the required welfare analysis for this firm?

51. A 2-input (x and z) and 1-output (y) firm uses a technology given by \( y = 10x^{1/3}z^{1/3} \). Input x is variable in the short term. Input z is only partially variable. If the firm wishes to change z from some starting level, it takes a full period before the change can be implemented. Initially, market prices have been \( (p, w, r) = (10, 4, 21) \) for \( (y, x, z) \), and the firm is in equilibrium with respect to these prices.

An exogenously caused policy change will immediately result in \( r = 16 \) if the policy is adopted.

Considering the immediately forthcoming 5 years for this single firm, we wish to compute an aggregate monetary value of this price change across all five years using the constant discount rate, \( d \).

a. Number and explain steps for a single procedure that calculates the needed value using computed supply and/or demand functions. Use supply and/or demand graphics to illustrate as appropriate.

b. Perform the analysis needed to calculate the value. Enumerate your work following the same steps you identified in part (a).

52. In a many-good economy, a policy under study will only alter the prices of two goods, \( x \) and \( y \). You are satisfied with using Marshallian measures of welfare change for this policy, and the aggregate demand functions have been determined to be

\[
x = \frac{2M^2}{p_x} \quad \text{and} \quad y = \frac{M^2}{p_y^2},
\]

where \( M \) is aggregate income. Is path dependence a problem for this setting?

53. Suppose that there is to be a simultaneous change in the price of \( X \) and the wage rate for a consumer with the following utility function (\( L \) is leisure): \( U = 2\ln(X) + 5\ln(Y) + 3\ln(L) \). The price of \( Y \) is unaffected. Is path dependence a problem? Explain.
54. Suppose that quantity demanded is a linear function of price with an undetermined and arbitrary intercept and slope. Determine a general formula for consumer surplus as a function of quantity (not price).

55. Estimate a variance and a confidence interval for a welfare measure you calculated in an earlier problem. The earlier problem must be one in which you statistically estimated some function.

56. Given \( U = 10y^{1/4}x^{1/4} \):
   
   a. Derive the expenditure function as well as the corresponding Hicksian demand functions.
   
   b. Assume \( p_2 = 4 \) and \( U^0 = 20 \). Use both the difference of expenditure functions as well as the area beside the appropriate Hicksian demand to calculate EV and CV when \( p_1^0 = 4 \) increases to \( p_1^1 = 9 \). Show that the two methods yield the same measure of change. What additional calculations are required to obtain \( \Delta S \)? Perform these and verify that \( CV < \Delta S < EV \) for this price increase.

57. A consumer with tastes for only two goods has the following indirect utility function. The goods are \( x \) and \( y \), and their respective prices are \( p \) and \( r \). \( M \) denotes income.

   \[ v(p, r, M) = \left( \frac{p^{0.5} + r^{0.5}}{2} \right)^{-2} M \]

   This person is about to experience an exogeneous change in \( p \) which is changing from 6 to 2. \( r = 2 \) and \( M = 280 \) will both remain unchanged. Calculate as many accurate Marshallian and Hicksian welfare measures as you can for this change. In what ways are your quantitative findings theoretically sensible in relation to one another?

58. Assume an individual whose preferences over goods \( x \) and \( y \) are represented by \( U(x, y) = xy^{1/2} \). Income is 48. Pre-policy prices are \( p_x^0 = \frac{1}{2} \) and \( p_y^0 = 1 \). Post-policy prices are expected to be \( p_x^1 = \frac{1}{16} \) and \( p_y^1 = 1 \) (that is, the price of \( y \) is unchanged).
   
   a. State whether compensating variation is greater than or less than equivalent variation and demonstrate mathematically.
   
   b. Also determine \( \Delta S \).
   
   c. Suppose you do not know the given utility function, but you do know the Marshallian demand function you obtained previously from it. Use this demand function to compute either \( CV \) or \( EV \) (state which).

59. Suppose ordinary demand for good \( x \) is estimated to be \( x = \frac{I}{p} \) where \( I \) is exogenous income, \( \eta \) is a constant (\( \eta > 0 \), \( \eta \neq 1 \)), and \( p \) is the price of \( x \).
   
   a. Compute income elasticity.
   
   b. Find the expenditure function, \( e(p, k) \), corresponding to this demand function.
   
   c. Suppose \( I = 1 \) and that a proposed policy will increase \( p \) from 1 to \( e(2.7183) \). As a function of \( \eta \),
      
      • Compute \( \Delta S \).
      
      • Compute \( CV \).
      
      • Compute \( EV \).
   
   d. What is the limit of \( \Delta S \) as \( \eta \) goes to zero?
e. What is the limit of CV as $\eta$ goes to zero?

f. What is the limit of EV as $\eta$ goes to zero?

g. What do (d)-(f) illustrate?

60. In a two-good, single-period world where both goods are priced, a consumer is able to buy as much of good 1 ($x_1$) as she likes, but the availability of the second good ($x_2$) is limited by government controls. The current situation is that $p_1=4$ and $p_2=2$ and the consumer has an income ($M$) which is fixed at $400$. Each consumer is allowed to purchase no more than 50 units of $x_2$. The consumer's preferences are fully accounted for in her indirect utility function which is given here:

$$v = \frac{1}{4}p_1^{0.5}p_2^{0.5}M.$$ 

The government is debating a proposal to reduce the $x_2$ "allowance" to 25 units due to the fact that consumption of this good causes more social harm (outside the consumer's scope of considerations) than previously thought. Prices will not change.

a. Compute obtainable Hicksian measures of welfare change for this proposal.

b. What is the Marshallian measure of welfare change?

c. Advise the decision-making body about your findings and the implications of these results for decision making.

61. A consumer with an income of $M$ is going to experience a $2$ rise in the price she pays for commodity $x$. Price is increasing from $10$ per unit to $12$. Her inverse demand for $x$ is given by

$$p = Mx^{1/n}$$

where $p$ is the price of $x$ and $n$ is a constant.

a. Determine and simplify a function expressing the change in consumer surplus ($\Delta S$) as a function of $M$ and $n$.

b. Determine and simplify a function expressing compensating variation (CV) as a function of $M$ and $n$.

c. Compute income elasticity.

d. For $M = 100$ and $n = 0.5$, compute $\Delta S$ and CV. Is your result surprising and why/why not?

62. As an emergency measure to address the State's budget crisis, Texas is proposing a stiff tax on beer purchases. The tax will raise the costs of a six pack from $2.56$ to $4.00$. The average adult Texan has an income of $Y = 100$ per week and has the following demand for six packs.

$$x = Y^{1/2}p^{-1/2}$$

where $x$ denotes six packs per week, $Y$ is weekly income, and $p$ is the price of a six pack.

a. Compute all appropriate Marshallian and Hicksian measures of consumer loss for the average adult.

b. The state economist contends that each dollar of tax revenues is used by the state to produce $2.30$ of benefits. Use this information to perform an analysis of the above tax proposal. What should Texas do?

63. The average individual in a 100-person community has the following demand for a local recreational good:

$$x_1 = p_1^{-2}p_2^2$$

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where the good $x_1$ has price $p_1$, $I$ is annual income (thousands), and $p_2$ is the price of some other good.

Currently, $p_1 = 2$, $p_2 = \frac{1}{3}$, and average per capita income is $2,000$ (use 2 in your calculations). City leaders are contemplating the desirability of increasing $p_1$ from 2 to 8 in order to cover increased production costs.

Calculate the appropriate Hicksian measure corresponding to the "status quo ante" test for the community impact of this policy change.

64. After some study, it has been determined that the preferences of a group of people (Group A) would cause them to behave as if they have the following collective expenditure function if there were markets for goods $x_1$ and $x_2$:

$$e(p_1, p_2, U) = 2\sqrt{p_1p_2U}.$$  

The $U$ term represents collective utility. Although $x_1$ is purchased in the marketplace at price $p_1$ (currently, $p_1 = $5), $x_2$ has been traditionally available to the group in fixed quantity without the opportunity for market exchanges. Up to this time, 16 units of $x_2$ have been available for free to the group. Now, a second group (B) has successfully pressured the government for a reapportionment of scarce $x_2$. As a result, Group A must suffer a decline in $x_2$ from 16 to 8, and their collective utility will be lowered from $U' = 200$ to $U'' = 100$.

a. What compensation would you argue should be paid to Group A?

b. If the computation of $\Delta S$ makes a contribution in answering this issue, compute it.

65. A professor is considering the possibility of writing a textbook in a subject area where no text is available. Because of the narrow field, he knows that no publisher will agree to publish the book. If he writes it, he will have to publish it himself, and he anticipates a 500-page book costing $20 per copy to print (printing costs only).

The professor wonders whether it will be worthwhile to write the text. He has decided to undertake this project if the monetary value of the text to his students is "large enough." No market data indicating the value of such a text is available, because no text exists. Because you are the top student in his class, he has insisted that you perform the needed analysis.

After interviewing some representative students, you determine that an average student benefitting from this book would behave as if he/she has the following utility function:

$$U = 5t^{0.2}d^{0.8},$$

where $t$ is the number of different textbooks in this field possessed by the student and $d$ is the number of compact discs the student purchases. Assume that a compact disc costs $10$ and that the average student has a $100$ budget for music and this text. While the professor has told you that there are no texts currently available in this area, you have decided that the notes he hands out to students for free are equivalent to 0.1 textbooks.

Assuming that students will only have to pay printing costs, what is the prospective net value of this text to students if there are 6 students this semester and 6 next year and 6 the following year and ...? What advice will you give to the professor and why?

66. On a two-century world, unconstrained behavior by each century's aggregated population has each applying symmetric preferences according to

$$U_1 = x_1^\alpha y_1^\beta \quad \text{and} \quad U_2 = x_2^\alpha y_2^\beta \quad \text{where} \quad \alpha >> \beta > 0 \quad (\alpha \text{ is much larger than } \beta).$$

1 denotes Centurion 1 (or C1): the community of all people living in the 1st century.

2 denotes Centurion 2 (or C2): the community of all people living in the 2nd century.
x₁ is consumption by C₁ out of its own production (q₁).

y₁ is gifted by C₁ to C₂, out of C₁’s production (q₁).

x₂ is gifted by C₂ to C₁, out of C₂'s production (q₂).

y₂ is consumption by C₂ out of its own production (q₂).

[The possibility that each Centurion has some regard for the other (β>0) is a reasonable consequence of two factors: mild altruism and some overlap in that some people are members of both C₁ and C₂. Notice that no one's utility is changed as a result of gifts received; assume this is a fact.]

An identical, decreasing marginal product production function applies for each period,

\[ q_i = m_i^k; \quad i=1,2; \quad k<1; \quad \text{with } m_1 + m_2 \leq M \]

where \( m \) is the only input. \( k \) and \( M \) are constants. The \( m_i \) are endowments – basically the legal or ethical division of the sole input across the two centuries.

Except for gifting, trade between C₁ and C₂ is impossible, so \( x_i + y_i = q_i \) and these 3 commodities necessarily have the same intra-century value \( (p_i) \). In general, \( p_1 \neq p_2 \) is expected.

a. For C₁ only and for arbitrary \( \alpha, \beta, k, \) and \( m_1 \), find the competitive equilibrium resulting from optimizing behavior by 2 agents. One C₁ agent is the producer of \( q_1 \), and the other C₁ agent is the aggregate buyer of \( x_1 \) and \( y_1 \) with the preferences stated above. The buyer/consumer is the only property owner. Neither agent exerts noncompetitive market power.

b. Accurately state the similar competitive equilibrium resulting for C₂.

c. Are the results of (a) and (b) efficient regardless of the \( m_i \), including the case where \( m_2 = 0 \) (which would occur if C₁ claims all \( M \) because C₂ is not present yet)? Discuss.

d. Under what conditions would the equilibria found in parts (a) and (b) result in \( p_1 = p_2 \) ?

e. If a social planner applied standard discounting tools to study optimal inter-century allocation, a social welfare function like the following might be applied. In light of your prior findings, what control variable could the planner employ to achieve maximum W, and what would the result be like? Please comment on the normative issues here.

\[ W = U_1 + U_2^{(1+\delta)^a} \]

with the discount rate \( d \) being somewhere in the 2-6% range.

67. \( U = R^2N \) is a utility function for a consumer of a rival good R and a nonrival good N. R is provided in the marketplace, and N is provided in fixed quantity by the government.

a. By solving the expenditure minimization problem determine the consumer's compensated demand for N at arbitrary prices and utility level.

b. The consumer has 18 units of income which she spends entirely on R. The price of R is 2 so she purchases 9 units of R. Initially, the government is providing 9 units of N. The responsible agency is considering a project which will increase N to 16. Assume the consumer's income and the price of R will be unaltered by the project. Using the results of (a) define and find two different Hicksian welfare measures for this change.

c. Suppose the project cost is 500 units and there are 100 identical beneficiaries of the project. Should it be built? Explain.

68. An individual consumes but two goods, x and y. Her utility function is

\[ U = 2x^{0.5}y^{0.5}. \]

x is purchased in the marketplace for a price of $8 per unit. The consumer spends all $800 of her income on x because y is provided by the government, and no y market exists. The government is considering an
increase in this consumer's allocation of $y$ from $y_0 = 4$ to $y_1 = 9$. The price of $x$ is assumed to be unaffected. Compute equivalent and compensating welfare measures for this change.

69. A project will increase the availability of a public good, $x$, from 1 to 4 units. Economic research has determined that each of the region's 100 people are represented by the following utility function,

$$U = x^{0.5}y^{0.5}$$

where $x$ is the public good and $y$ is a composite of all other goods. The price of $y$ is 1, and each consumer can spend all 100 units of his income on $y$ because $x$ is provided at no charge to the consumer.

The decision-making committee has ruled that projects such as this can only be accepted if they satisfy both Hicks and Kaldor criteria. If the project is to be accepted, what is its maximum tolerable cost?

Alternate Question: Assuming the project will be fully operational for exactly ten periods in which the given details are replicated: if the project is to be accepted, what is its maximum one-time (period 1) tolerable cost? The project has no costs during the last nine periods.

70. For her dissertation work a student has carefully analyzed the demand for recreational fishing. She has determined that the average fisher has the following Marshallian demand for fishing:

$$f = \frac{m}{2p}$$

$f$ is the number of fish caught on a weekend. $m$ is thousand $\$ of annual income, averaging $60 thousand per fisher. $p$ is the price per caught fish. [Although a market does not exist for $f$, the Ph.D. student has applied an interview methodology to get fishers to reveal their actual demand responses. The methodology is 100% accurate.]

A Judge has previously ruled that (1) every fisher has the right to take 20 fish per weekend but no more than that, and (2) every fisher must always pay 50 cents per caught fish to the state agency that stocks rivers with fish. Enforcement of these rules is perfect and costless.

a. The Environmental Group prefers a lower per person catch than 20 fish per weekend. Based on the available information, how much would The Group have to exactly compensate the average fisher to lower the number of caught fish to 10 per weekend? The $0.50$ price will still apply for the 10 fish.

71. Together, the residents of Ourtown behave as if they maximize aggregate utility given by

$$U = H^3G^6$$

where $H$ is annual expenditures on all private household goods and $G$ is the annual flow of public services. Aggregate annual income in Ourtown is $100$. In this case $G$ comes solely from publicly owned, nondepreciating capital (e.g., swimming pools, parks, zoos, roads) that have no operating costs. $G$ is currently 2 units. The local government sometimes makes investments to increase $G$ by adding to this public capital. Except for these investments, all income is spent on $H$ which has a price of $1$ per unit. Ourtown has no existing debt, and it can borrow any funds it wishes.

To guide decision making on the attractiveness of alternative public investments, Ourtown employs the Kaldor criterion, a 6-year planning horizon, and a 10% discount rate (its cost of borrowing). Ourtown government is considering a one-time investment costing $400 that will immediately and permanently raise public service flows from 2 to 4 annually. Perform the appropriate analysis to see if this investment should be undertaken.

72. The rural people of a poor and remote village are without electrical power because they are located far from such services. Nor can they afford to provide themselves with electricity through either individual or collective means. The traditional national policy has been to provide a limited number of free batteries to each household. This policy gives each household one (1) unit of electricity.
An alternative policy is being considered. Small photovoltaic (PV) systems have an annualized cost of $100 and are capable of running a few light bulbs and small appliances several hours each day. The variable cost of operating such a system is $0. A government agency is considering whether to provide each willing household with such a system at a subsidized price of $10 per year.

Each household possesses an annual income of $20 to be spent upon food (F) and electricity (E) on the basis of the following utility function:

\[ U = F \cdot E. \]

The price of F is 1 and will be unaffected. By paying the $10 lump-sum fee annually, a household can increase its E consumption from 1 to 10. Perform a welfare analysis for the government agency considering the purchase of a single PV system for a representative household. The selection of a potential Pareto criterion is up to you, but you must apply it exactly.

73. A friend of mine estimated a system of demand equations and found that the corresponding utility function was

\[ U(x) = 2\sqrt{x_1} + x_2 \]

The government is considering a policy that would lower the equilibrium price of \( x_1 \) from $4 to $2. The supply of \( x_2 \) is perfectly elastic at \( p_2 = $1 \), and income is $10,000. My friend tells me that the government policy should be adopted, provided the cost of implementing the policy is "sufficiently low." Do you agree with my friend? If not, why not; if so, how low is "sufficiently low?"

74. Individuals in the lowest two income classes have mean incomes of 9 and 18. These incomes are inadequate for providing subsistence standards of living. As a result, these people have poor health and die earlier than is normal. Instead of providing income supports, society is considering food subsidies for members of these two income classes. The policy proposal is to issue nontransferable food coupons which entitle the recipient to a 40% discount on all food purchases (0.6 times old food price = new price). The government will reimburse merchants for all coupons exchanged in this manner. Assume this policy will not alter equilibrium food and nonfood prices. Let \( x_1 \) represent nonfood items and \( x_2 \) represent food. The numeraire is \( x_1 \) (\( p_1 = 1 \)), and \( p_2 = 5 \). Utility functions are equivalent across all individuals:

\[ U = 4x_1^{1/2}x_2^{1/2} + x_2 \]

a. Perform the analysis necessary to obtain Kaldor and Hicks measures of welfare change for individuals in each group.

b. Identify the fiscal effects on government.

c. Do you conclude that this policy is worthy?

d. Do your conclusions in part c change as a result of the following statement?

"Poverty programs are to be assessed from the perspective of a utility basis selected so that all people have the right to a subsistence standard of living, such as is provided when \( x_1 = 2 \) and \( x_2 = 8 \)."

Multiagent Welfare Analysis

75. A regional industry is composed of 16 y producers, and a representative firm in this industry has the production function \( y = 2x^{1/2}z^{1/4} \). The regional supplies of inputs x and z are given by \( p_x = 4 \) and \( p_z = 16 \). Total demand for the output of this industry is given by \( y_D = \frac{1}{p} \) where \( p \) is the price of y. A "price support" policy currently exists for y. According to this policy, \( p = 5 \).

There is a policy proposal to be evaluated. This new policy would establish a revised price support policy in which \( p = 7 \).

a. Perform all practically computable policy analysis. Also, as a check on your calculations, compute the producers welfare impact in at least two ways, neither of which computes \( \Delta \pi \) using a profit equation.
b. Summarize your findings and provide a discussion of these numbers so that a decision-making committee can properly understand these results.

76. In order to perform analysis for a proposed policy reform involving two related markets, price-quantity and income data has been collected for 5 recent periods and 10 representative individuals. This data is tabulated below and is downloadable at ron-griffin.tamu.edu/agec636HWdata.txt.

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Prices and income are expressed in $, and they have been adjusted to real terms (in today's $). The stated prices are best estimates of prices faced (with tax and subsidy included) by the average consumer during each period.

The policy proposal is to simplify government operations by removing both a 5¢ per unit subsidy on purchases of \( X_1 \) and a 5% sales tax on purchases of \( X_2 \). Analyze this proposal for its welfare impacts and give a clear recommendation. You may assume that these commodities are supplied with perfect elasticity. Because this is not an econometrics course, using OLS estimates in your analysis is okay. As always, be sure to provide summary information regarding your statistical analysis.

77. Based on the results of several econometric studies, the price elasticity of demand for \( x \) is thought to be \(-0.5\), and the price elasticity of supply is \( 1.0 \). Average annual consumption of \( x \) is approximately 1,000,000, and \( x \) has been trading at $5 per unit.

a. Assume demand and supply functions are linear. Determine demand and supply functions that would produce the above scenario.

A price floor of $6 will be established by a proposed policy. Two versions of this policy are under consideration. Policy A contains only the price floor. Policy B contains the price floor and a guarantee that an agency of government will purchase all surplus production at the floor price.

b. What are initial quasi-rent and consumer surplus? What are subsequent quasi-rent and consumer surplus for each policy? What are the changes in these welfare measures for each policy? Discuss your interpretation of which policy should be adopted.

c. Assume the supply function is linear but demand exhibits constant elasticity at all price and quantity levels. Redo parts (a) and (b) of the above analysis under this new demand structure.
d. Assuming that (c) represents the true case, how good or bad does part (b) approximate "truth". Discuss. Can you envision any guidelines for applied work?

78. A public good (x) with total production costs of \( C = 600 + \frac{1}{2} x^2 \) is currently supplied by government in the amount \( x=220 \). Consumers of x can currently use the good for free. Production of x is financed by a per unit tax on the sales of private good y. You may assume the tax is currently $1 per unit of y. The industry supply (pretax) of y is given by \( y_s = -\frac{799}{2} + \frac{1}{2} p_y \) and the aggregate demand (pretax) for y is \( y_d = 50000 - \frac{1}{2} p_y \).

There are two types of public good consumers, a and b, and there are 12100 consumers of each type. Their individual demands, for each consumer, are \( x_a = \frac{1}{p} \) and \( x_b = \frac{3}{p} \).

a. Calculate the tax revenue and the y-market price and quantity results under initial policy.

For b and c below: A new technique called Dude has been invented for achieving exclusion of x users. If Dude is implemented, the y tax can be eliminated, and pricing of the x good can be used as the financing instrument.

b. Calculate x and y price and quantity results assuming that Dude is implemented and x is produced at an optimal level.

c. In light of its aggregate welfare effects, what is Dude worth to society? Is this surprising in any way?

79. The economic downturns which are part of business cycles are especially hard on marginal enterprises. As a result, recent recessionary tendencies have threatened the economic viability of numerous farms, and most farm operators have seen lowered prices for their products. Consequently, some farms are going bankrupt. To address this social problem a legislative committee is proposing to benchmark 2009 farm output prices against 2000 prices. The committee's proposal is as follows:

Policy: Any unprocessed, farm-produced commodity (such as corn, oranges, or beef) having a current price more than 10% below its average 2000 price will be traded at a controlled price which is exactly 10% lower than its 2000 price. This policy will expire at the end of the calendar year.

Your task is to analyze and discuss this policy in a manner that will be helpful to decision makers. Time is short because a vote on the proposal will occur soon. Assume two types of farm products. "Type A" products are directly affected by this policy because their prices are too low relative to 2000 prices. "Type B" products have 2009 prices greater than 0.9 times their 2000 price. Perform some analysis using the following information and explain your findings to legislators. Note any expected omissions in this work. Make a voting recommendation to voters and justify it. Baseline information is as follows:

In the remainder of 2009, 10 million tons of type A commodities are expected to trade at an average market price (weighted of course) of $5 per ton. The 2000 price for these goods was $8 per ton. Elasticity of demand for the type A basket of goods is -0.5, and its supply elasticity is +0.5. For perspective, the value of type B goods traded during the rest of 2009 is expected to be $40 million.

80. The government of a developing nation is trying to decide whether to invest in a cost-reducing technology for one of the basic food crops. The demand (\( \varepsilon \)) and supply (\( \eta \)) elasticities for this food have been estimated to be -0.5 and 2.0 respectively. The price (P) is $2, and 5,000,000 units are produced and sold each year. The amount of investment required would be $500,000 per year. The new technology would have the effect of shifting out the supply curve (S) by about 10% and would not affect the demand curve (D). In other words, for any price, about 10% more would be supplied than would have been supplied without the new technology. There are no fixed production costs in this setting. Assume the following identity relating the percentage change in equilibrium price to terms indicated previously where both supply and demand functions are presumed to exhibit constant price elasticity:
\[
\%\Delta P = \frac{1}{\varepsilon - \eta} \left( \%\Delta S - \%\Delta D \right) .
\]

a. If the investment is made, what would be the effect on the market price and quantity for this food?

b. Using the change in consumer surplus as the measure of policy benefits and assuming no other information is available, would you advise making the investment or not? Explain your reasoning and assumptions. [You may approximate the change in consumer surplus as a trapezoidal area].

c. The analysis would obviously be more satisfactory if welfare effects for crop producers could be incorporated. Do these additional impacts alter the recommendations forthcoming from part b.

d. If you could gather additional information, what, if anything, would you like to have? Might there be other considerations that would cause you to reverse the recommendations you provided in part c?

81. Because nation "US" cannot believe that its future people are as important as "us" current people, it is unable to establish policies that include externality costs in fossil fuel prices (e.g. gasoline prices). But the nation might enact some policies promoting alternative energy, as long as fossil fuel consumers are affected only indirectly. A 4-year price support policy affecting the sale of solar panels (SPs) to households is under consideration. Policy objectives include encouraging the domestic production of SPs and reducing worldwide, energy-caused pollution. Any governmentally purchased SPs will be donated to island countries to assist poorer people being subjected to sea level rise while also lowering their fossil fuel emissions.

Without any US government interventions the representative US consumer behaves as if they have a separable energy budget B for buying only fuel (F) and SPs (S) with preferences given by

\[ U_h = F_h S_h^{0.5} \quad \text{and} \quad B=12. \]

There are 100 million consumers. Assume a partial equilibrium situation in which the price of F is always 1. Commencing in the initial year (t=0) US SP producers behave as if they have the aggregate total variable cost function

\[ TVC = \frac{4-t}{128} S^2 \quad t = 0,1,2,3. \]

The support policy promises that for 4 years starting in year 0 the support price in all 4 years will be fixed at 20% larger than year 0's expected competitive price. In return, all SP producers are required to commit 50% of their increased profits to investments in advanced production processes. A consequence of this requirement is variable costs should decrease each of the 3 years after the first. This is caused by capital investments and improved knowledge (and is displayed in the cost function). The support policy will be removed at the end of t=3.

a. Imagine a table with Years 0-9 in the first column. The rest of the table is empty except for column headings. These column headings identify every effect the nation might regard as important to the analysis of this policy. List and individually explain these headings. Mention noteworthy considerations as needed. Ideally, your agency's analysts will be able to quantify all entries needed to complete this table (but not today).

b. Make some progress in quantifying entries for your imagined 10-year table. Explain your choices and limitations. Contributing results for multiple columns is more important than contributing results for multiple years, but all achievable estimates are important. Show all the needed work clearly.

82. Public desire for preserving the tradition of family-owned farms has resulted in the adoption of policies that are advantageous to these farms. Assume that these policies are administered at various governmental levels and that these policies do not directly address farm product prices. Instead, these policies provide tax advantages for qualifying farms, implying that tax payments by these farms are lower (per unit of product and per acre) than those by larger "corporate" farms. As a consequence, 50% of the nation's farm acreage is currently in family-owned farms and 50% is in corporate farms. Were it not for the tax relief policies, all
family-owned acreage would have been converted to corporate acreage due to the lower production costs of larger, more specialized farms. Assume that land is fully homogeneous.

Marginal costs for producing foodstuffs, \( y \), across all current corporate farm acreage is \( MC_c = 0.2y_c \). [This is marginal costs across all corporate farms; it is not marginal cost per acre or per farm.] Similarly, marginal costs for producing foodstuffs, \( y \), across all current family farm acreage is \( MC_f = 0.4y_f \). Both of these marginal cost relationships are currently observed, after tax functions. Aggregate demand for foodstuffs is \( y = 100 - 2.5p \) where \( p \) denotes price.

a. Given the above information, provide some quantitative results about the social costs of our policies encouraging family farming.

b. How complete are your results? That is, can your findings be relied upon for a recommendation about the desirability of these tax relief policies? Why/why not?

83. The government is considering policy mechanisms for encouraging the production of timber, because forests have the desirable property of tying up carbon, an important element of CO\(_2\), which is the primary global-warming gas. Because timber products such as pulp or firewood may release their carbon relatively quickly, we are especially interested in timber that will be utilized in long-lived wood products such as furniture and housing.

To encourage the right kind of timber production, a Bill is being sponsored to create a subsidy of $11 per million board feet (mbf) of sawn timber. Presubsidy conditions are that 500 mbf of sawn timber are being sold annually at a market-clearing price of $100/mbf.

Supply and demand parameters are suggested by two recent studies. An econometric paper places the elasticity of derived demand for sawn timber at \( -2 \). A second paper indicates that the average sawn timber producer experiences a $10 increase in marginal costs for every additional mbf produced. There are 10 such producers.

a. Perform a welfare analysis for the consumer and producer impacts of the Bill. Assume that demand and supply functions are both linear. Include a clear sketch of the functions and welfare areas.

b. Given that this subsidy will induce price repercussions in other markets, how would you interpret your welfare results of part (a), and what suggestions would you offer for additional analysis?

c. What are your recommendations regarding the attractiveness of this policy?

84. Local government regulates the price charged by the only local provider/producer of product V, which is a basic commodity used by all households (like natural gas, electricity, or garbage pick-up). The producer is requesting that it be allowed to raise V's price from $400 to $500 per unit, because of a recent rise in the price of a crucial production input, U, from $40 to $60. [V producers don't demand enough U to affect its price noticeably.] Recently, the quantity of V demanded by all local households has been 240 per year at the regulated price, and the producer has been using 1200 units of U. Your task will be to perform a policy analysis of the provider's request.

Although the price of V is locally invariant, it varies from locale to locale. An econometric study of demand, using data from many regions, suggests that the price elasticity of V demand is about \( -0.5 \). Recently, a study commissioned by U suppliers indicated that the partial-equilibrium price elasticity of U demand by V suppliers is about \( -0.5 \) as well. Furthermore, there is other evidence that a $10 rise in the price of V has tended to increase U demand by 45 units.

a. Compose a quantitative policy analysis for the local government using the above information. Presume that demand/supply functional forms in the U marketplace are linear (i.e. \( X = a + bY \)) and that demand/supply in the V marketplace are constant elasticity forms (i.e. \( X = aY^b \)). As the baseline, prepolicy situation, presume \( (V_0, U_0, p_v^0, p_u^0) = (240, 1200, 400, 40) \). Clearly draw and label all welfare measures needing computation. Calculate welfare measures.
b. Discuss your monetarization analysis in relation to the welfare measures desired here. What policy recommendations emerge (or should emerge) from your work? Would you approve, disapprove, or modify the request or would you be inconclusive? Why? If V expenditures are significant budget items for some households, what additional analyses/considerations should be contemplated?

85. Nation A has what it considers to be a serious trade imbalance with Nation B. Nation A has been attempting to deal with this problem using quotas upon the imports of specific goods. As an economist working for Nation A you are charged with the economic analysis of a change in one of these quotas. An import quota of 10 units of good x is proposed to be relaxed to 20 units. Both quotas are binding constraints upon the import which is a final consumption good. Excess demand by Nation A consumers for x produced by Nation B is estimated to be

\[ x = \frac{200}{5+p} \]

where p is the price of imported x.

a. Would the additional knowledge of excess supply of x by country B assist your work? Why?
b. Can you employ the given information to find an exact measure of welfare for the policy change? Explain.
c. Other quotas are also under consideration for modification. Is this relevant for your work? Why?
d. Perform the needed analysis to the best of your ability.
e. Do you recommend adoption of this policy change? Why?

86. The Z market is composed of 12 homogeneous firms that use inputs x and y to produce z according to the following production function:

\[ z^i = 10x^{1/3}y^{1/6} \]

for \( i = 1, 2, ..., 12 \).

The supplies of x and y are perfectly elastic at prices \( p_x = 4 \) and \( p_y = 2 \). In the absence of any market-influencing policy the total market demand for z is given by

\[ z_d = \frac{25}{p} \]

where p is the market price of z.

a. Compute all aspects of the market equilibrium when behavior is perfectly competitive.
b. Suppose that government acts to stimulate z production by subsidizing the purchase of z. The subsidy shifts the market demand function to \( z_d = \frac{25(1+s)}{p} \) where p is the producer price of z and s=0.44. Find the new equilibrium, the change in consumer surplus, and the change in producer surplus that is caused by this policy.

87. The market supply of commodity x is given by \( p = 2(1+t)x \) where p is the price of x to consumers and t is the tax rate. Demand for x comes from 10 equivalent consumers, each having an income of $25 and the utility function \( U = xy^4 \) where y is a composite of all other goods and is the numeraire good. Suppose the tax rate goes from 0% (t=0) to 21% (t=0.21).

a. What is the change in consumer surplus for this tax increase?
b. What is the change in quasirent for this tax increase?

Alternative question (instead of a and b): Provide a complete numerical assessment and discussion concerning the social desirability of this tax change.
Four identical farms each use chemicals, c, and a cost-minimizing bundle of other inputs, x, in the production of a crop output, y. The unit price of c is $2, and x costs $1 per unit. Each farm's production function is

\[ y = (1+c)^{1/2}x^{1/4}. \]

Market demand for Y (the sum of all four farms' output) is \( Y = \frac{64}{P} \) where \( P \) is the price of Y. Because of food safety and water pollution concerns, a ban on the use of chemicals has been proposed. In terms of welfare measures, what are the impacts of this policy on producers and on consumers? Sketch and determine mathematically.

One hundred consumers each have the following demand for energy: \( E^d = \frac{10}{p} \), where \( E^d \) is the individual's demand for energy at price \( p \). The industry's marginal cost of producing energy is \( MC = \sqrt{E^s} \) where \( E^s \) denotes energy supplied. The industry operates in the market according to this cost structure. Unfortunately, these costs omit certain external costs of energy production such as pollution, global warming, and the depletion of nonrenewable resources. These additional social marginal costs are measured by \( ASMC = \frac{5}{2}E^s \). Because of this problem, the government is planning to raise the current market price of energy by $0.50. The government will take this $0.50 as a tax. Determine the appropriate monetary measures of the social impact of this policy and justify your work as you present it. How would you advise government? How would you improve this policy?

A city owns a large stadium used only by a sports team to play 10 games each year. Whereas the stadium is publicly owned, the team is privately owned. There is one such team in the area. The stadium is used only when the team hosts a game played against a team from another city. Stadiums do not depreciate, have no operating costs, and this one can seat 60,000 people. Every seat in the stadium provides the same experience; seats are perfect substitutes for one another. It is not possible to expand the capacity of the current stadium.

Ticket sales are entirely managed by the team's owners, who are profit motivated. No revenue is received by the city. All of the team's costs are fixed costs, in the sense that costs are independent of the number of seats used during games. That is, costs are the same regardless of attendance. Demand is sufficient to make the team profitable.

Ticket demand for each game is given by \( q_0 = 120,000 - 1200p \).

Due in part to the team's increased popularity (already reflected in the \( q_0 \) demand function), the city is considering a proposal to demolish the stadium and replace it with a more modern facility. The new stadium will hold 96,000 people and will only take a few months to construct. No games will have to be cancelled. With a more modern facility the sporting experience will be better for the audience, so the demand function is expected shift to \( q_1 = 126,000 - 1200p \). Again, the new seats will be perfect substitutes for one another.

Stadium replacement will cost $100 million and the city plans to pay this amount by initiating a 5% sales tax rate on all locally sold goods (except sporting event tickets). Once the stadium costs are retired, the sales tax rate shall be returned to 0%. The sales tax base consists of $250 million of goods sold annually. [Let's think of this as 250,000 units of a good priced at $1000 each.] Supply and demand elasticities for this aggregate good are estimated to be \(+0.5\) and \(-0.5\) respectively.

a. Perform and interpret quantitative analysis to assist city leaders in making a decision about the stadium proposal. Do what you can and explain the rest. If additional information is necessary for completing your analysis, explain the needed elements and how (quantitatively) they are to be used. Include interpretation and project advice for when you complete all parts of your study.
A gas company is the sole provider of natural gas (G) to industry and homes in a region. Homes tend to use G for heating and cooking. Due to the company’s status as a natural monopoly, its product prices are regulated by a government board. Because of the extensive network of pipes needed to deliver G to homes, the cost of serving homes is greater than the cost of serving industry (for equal amounts of gas).

You are charged with study of a pricing policy change for household gas use. Current policy is that the company can set the price of G to homes no greater than A% of the price that would prevail if competitive market conditions were present, and the gas company must satisfy all demand at that price. The proposed policy change is to change A from its current level of 90 to 80.

Aggregate household demand for G is given by \( G = \frac{1}{10p} \) where I=750 is aggregate household income and p is the price of G. The company’s regional cost of providing gas to homes is given by
\[ \text{Cost} = 0.02G^2 + 2G + K \]
where K is the company’s fixed costs.

a. Perform the quantitative welfare analysis needed for the proposed policy change. Present your analysis and conclusions to the regulating board.

b. Indicate the assumptions and problem areas of your analysis. Are there available corrective measures which would eliminate some assumptions or problems? [If you’ve performed such a great analysis that it’s genuinely difficult to answer part (b), you will not be penalized.]

Farms commonly borrow money to pay cropping expenses at the beginning of the season. In a normal year a region’s 10 farms each produce food (F) applying the same technology that uses borrowed bank funds (B) and labor (L) according to \( F_i = 10B_i^{1/3}L_i^{1/3} \) for \( i = 1,2,\ldots,10 \).

These farms behave competitively in the 3 markets. Food demand is given by \( F_D = 888p^{-0.357} \) and is only supplied by the 10 farms (no imported or exported food). r, the price of B is competitively determined by the national market, and this market is normally cleared by \( r = 1.1 \). The price of labor is \( w \), and the supply of labor is national, highly mobile, and elastic at \( w = 10 \).

This is not a normal year. Here are the initial conditions for this year: The technology is unchanged. A global macro problem is causing regional B availability to be constrained below usual purchases by the 10 farms. Furthermore, the national government is prohibiting any rise in r as would normally occur to equilibrate this financial market. Only 150 units of B are available, and an emergency national policy is restricting r to be no greater than 1.1. So each farm is entitled to buy 15 B at a price of 1.1.

a. Given this year's abnormal conditions, what will be the equilibrium results of the food and labor markets?

Another market intervention is being considered, and you are to perform a welfare analysis of its effects only, given the abnormal conditions and programs that are in place. The government is considering making an additional 21 B available to each farm at the controlled price of \( r = 1.1 \). The government’s cost of obtaining this capital is unspecified.

b. What are the welfare effects of this latter policy on each potentially relevant agent group in this abnormal year? Provide the appropriate graphs, calculations, and explanations. Accuracy, completeness, and presentation matters to your audience.

A technological breakthrough in the production of commodity x promises to increase the supply of x dramatically. In fact, the OTA (Organization for Technology Adoration) estimates that the market supply function will shift from \( x = 10p_x \) to \( x = 160p_x \). Your task is to perform the analysis needed to understand the impacts of this change. Assume that the affected society consists of five (5) identical individuals, each of whom has preferences represented by \( U = xy \) where y is some other good. Assume that \( p_y = 4 \), and \( p_y \) is unaffected by the breakthrough. Income is 64 for each individual.
a. Determine how well the change in consumer surplus (ΔS) approximates compensating variation (CV) and equivalent variation (EV) by computing all three of these welfare measures.

b. What is the change in quasi-rent (ΔR) for the producers of x?

c. How would the results of (a) and (b) above change if there were more consumers? Fewer? What implications do you derive from these results for the issue of who should fund research?

94. A region is populated by individuals each having the utility function \( U_i = f_i(R_i, N_{i1}, N_{i2}) \) where \( R_i \) is the only rival good, and \( N_{i1} \) and \( N_{i2} \) are both nonrival and nonexclusive goods. \( N_{i1} \) is the amount of water recreation type 1, and \( N_{i2} \) is the amount of water recreation type 2. \( N_{i1} \) represents natural environments which permit stream-based fishing and, in certain areas, canoeing when water levels are sufficient. \( N_{i2} \) represents reservoir environments with lake-based fishing, year-round boating, and skiing.

Current levels of these two recreation types are \( \bar{N}_{i1} \) and \( \bar{N}_{i2} \). \( N_{i2} \) environments can be increased by constructing projects which decrease \( N_{i1} \) environments. Such conversions are irreversible. Of course, reservoir projects induce a wide range of nonrecreational costs and benefits.

The regional authority is contemplating a new project which will increase \( N_{i2} \). As part of the CBA, the decrement in \( N_{i1} \) and the increment in \( N_{i2} \) must be valued. Answer the following questions separately.

a. Assuming \( f_i \) and income are known for all individuals, how would you conduct the valuation? Provide appropriate theoretical detail and illustrations.

b. Assuming utility functions are unavailable, how would you conduct the analysis? Assume there are no funding limitations for your research. Justify your answer by identifying alternatives and making a rational selection.

c. Income varies greatly within the region. This may be a factor because participation in \( N_{i1} \) recreation generally requires substantially more capital (a boat and other equipment) than does \( N_{i2} \) recreation. How might you deal with this observation in the CBA?

95. As a means to address a long-standing pollution problem, government is going to levy a 40% tax on all fertilizer purchases by crop producers. Fertilizer is presently untaxed. Your overall task is to perform a complete analysis of this policy by obtaining monetary measures of welfare change for all impacts of this policy and to use these findings to assess the merit of this policy.

Background:
Collectively, consumers as a group gain utility from crop consumption and experience disutility from fertilizer-induced pollution according to

\[ U = C^{2}F^{-1} \]

where \( C \) is crop consumption and \( F \) is the amount of fertilizer employed by producers. The consumer group has an income of $64 which is entirely spent upon \( C \).

\( F \) use is determined by crop producers who behave as if they face the following aggregate production function:

\[ C = 8F^{0.25}L^{0.25} \]

where \( L \) is labor input.

The industry supplying fertilizer has the following supply function: \( p_F = F \), and labor supply is \( p_L = 4 \).

Tasks:

a. Select a criterion or a set of criteria that you will employ to analyze this policy. [Your choice(s) is(are) constrained to those that rely upon monetary welfare measures.] Justify your selection.
b. List the welfare impacts that you must evaluate in order to perform a complete analysis. Provide a rough illustration of each one.

c. Perform the numerical work needed to obtain all relevant welfare measures. Unless you note otherwise and explain your reasoning, it will be assumed that your calculations follow the illustrations you performed for task b.

d. Summarize your empirical findings.

e. Present your recommendations. Discuss all limitations that may apply to your work or recommendations.

96. A certain Texas ranch has the following demand for agricultural labor: \( L^D = 17 - w \) where \( w \) is the wage rate. The local supply of agricultural labor \( (L^S) \) consists of Texan labor \( (L_T) \) and one illegal alien \( (L_A) \). The supply of Texan labor is \( L_T = \frac{w}{2} - 2 \), and the alien's utility function is \( U_A = M_A e^{-L_A} \) where \( M \) denotes income. Labor is the alien's only source of income.

Policy Proposal: to increase the efforts of the U.S. Immigration Service (costlessly). This policy will result in transporting the alien back to his country of origin.

a. Using the Marshallian concepts of supply and demand, perform the quantitative welfare analysis needed to identify impacts upon all affected parties. According to the potential Pareto criterion, does this appear to be a desirable policy?

b. Because the proposed policy has a nonmarginal effect on the illegal alien, \( \Delta S \) (or \( \Delta R \)) may be a poor approximation of equivalent or compensating welfare measures. Redo this part of your analysis by calculating both of the Hicksian measures. What do you conclude?

c. The issue of whether or not the illegal alien "counts" in this problem can be quite important. Comment on this issue. How would you handle this problem?

d. A more vigorous program to deport illegal aliens would have a number of additional impacts and considerations. (i) Domestic agricultural workers may belong to low income classes. (ii) The presence of illegal aliens may burden the educational system because of the strong need for bilingual education. (iii) The U.S. may possess a traditional responsibility to accept all immigrants. (iv) The supply of particular farm commodities may be very negatively influenced. Clearly, the identification problem is a difficult problem for this policy. How, specifically, would you handle these four concerns in a social impact analysis? In the final analysis, how important would the quantitative research done above be? Why?

General Equilibrium Welfare Analysis

97. Consider a simple vertical market structure in which industry \( i \) produces \( y_i \) using \( y_{i+1} \) as its only input. The product, \( y_i \), is sold only to industry \( i+1 \). Profit functions for three of these industries are

\[
\pi_3 = 8 \frac{p_3^2}{p_2} \quad \pi_4 = 8 \frac{p_4^2}{p_3} \quad \pi_5 = 8 \frac{p_5^2}{p_4}
\]

The supply function for \( y_2 \) is \( p_2 = 1 \), and the demand function for \( y_5 \) is \( p_5 = 64 \).

a. Find the competitive equilibrium. This is a touch tricky due to the multiple, interrelated markets. Sketch supply and demand curves and the CE for all four markets.

b. New regulations are causing the price of \( y_2 \) to double to \( p_2 = 2 \). How does everything change in response to this policy?

c. Compute welfare measures for all five industries. Comment on this policy.
d. **General equilibrium:** In part a you have hopefully seen and dealt with the fact that the equilibrium \( p_4 \) depends upon \( p_3 \) (and vice versa). In fact, \( y_4^s = y_4^d \) produces an equation specifying \( p_4 = f(p_3) \).

Substitute this relationship into your function \( y_3^d \) in order to eliminate \( p_4 \). The revised demand function can be called \( y_3^{d*} \) to indicate a general equilibrium demand function. It is general equilibrium because it is a function of own price only and the feedback of \( p_3 \) changes through \( p_4 \) is built in. Redo that part of (c) where a welfare measure was computed for industry 4. This time use \( y_3^{d*} \) and merely integrate this function across the \( p_3 \) price change. How does the new measure relate to your earlier measures of part (c) and why? Consult chapter 9 of Just, Hueth, and Schmitz if you have difficulty interpreting these results.

98. *This problem extends problem 45 to the setting depicted by Figure 9.4 in Just, Hueth, and Schmitz.* Let there be twelve identical firms of the type described in problem 45. These firms compose Industry A which is the only consumer of commodities \( X_1 \) and \( X_2 \) and the only producer of output \( Y \). The demand for \( Y \) is given by \( P^4 = \frac{4}{5} \). The supply of \( X_1 \) is given by \( X_2 = 0.329w_2^{1.3016} \). Initially, the supply of \( X_1 \) is given by \( w_1^0 = 9 \); subsequently, it is \( w_1^1 = 16 \).

a. Determine the industry's partial equilibrium, initial and subsequent demand functions for \( X_1 \) and \( X_2 \) (four different demand functions). Are \( X_1 \) and \( X_2 \) complements or substitutes?

b. Sketch the results of part (a) together with the supply functions to reconstruct Figure 9.4.

c. Use the partial equilibrium results to compute the change in quasi-rent for price changes affecting Industry A as determined in the \( X_1 \) market.

d. Use the partial equilibrium results to compute the change in quasi-rent for price changes affecting Industry A as determined in the \( X_2 \) market.

e. Determine the change in quasi-rent for the industry supplying \( X_2 \).

f. Determine the general equilibrium demand function for \( X_1 \).

g. Use the result of part (f) to compute the change in quasi-rent when \( w_1 \) changes from 9 to 16.

h. Compare the results of parts (c), (d), (e), and (g).

99. Producers of \( y \) use a number of inputs, including \( x \), in a manufacturing process. You could compute \( \Delta R \) for a policy-induced increase in \( p_y \) using either of the following supply specifications:

(A) \( y = \alpha_0 + \alpha_1 p_y \) \hspace{1cm} (B) \( y = \beta_0 + \beta_1 p_y + \beta_2 p_x \)

Interpret the following three welfare measures. (i) \( \Delta R_A \); (ii) \( \Delta R_B \); and (iii) \( \Delta R_A - \Delta R_B \).