JOURNAL OF ENVIRONMENTAL ECONOMICS AND MANAGEMENT **14**, 41–53 (1987)

Environmental Policy for Spatial and Persistent Pollutants[1]

RONALD C. GRIFFIN

Department of Agricultural Economics, Texas A & M University, College Station, Texas 77843

Received October 30, 1984; revised August 6, 1985

Traditional economic models of alternative pollution policies notwithstanding, all discharges are persistent to some degree (assimilation is not instantaneous), and their distributions vary spatially. Utilizing an optimal control framework to capture the dynamics of persistence, the efficiency of economic incentives and regulations are juxtaposed when the goal is to obtain arbitrary environmental standards at least social cost. For generality, the considered pollutant is regarded as spatially variant, and standards are allowed to differ among regions. Theoretically optimal policy parameters are derived. As in the case of spatial, nonpersistent discharges, the property of persistence alone is demonstrated to invalidate the typically maintained economic advantage of price-guided policies over regulatory policies.

Tietenberg [19, 21] is generally credited with being the first to recognize that spatial variability in the regional distribution of pollution can invalidate the efficiency properties of uniform effluent charges. This fact has been noted by Mäler [9], reviewed by Baumol and Oates [1, pp. 144–147], and extended by Henderson in the case of air pollution [4]. Tietenberg's analysis [19] is the most rigorous and demonstrates that incentive policies for spatially differentiated externalities are considerably more "information intensive" than those for nonspatial externalities [19, p. 12]. Mäler's approach is useful in that he begins to demonstrate that the economist's perception of the relative merit of price-guided policies (subsidies, taxes, charges, markets) and quantity-guided policies (standards for emissions, ambient environmental quality, or production practices) is altered whenever the externality varies spatially. Mäler's analysis also suggests that the superiority which is usually attributed to price-guided policies hinges on their reduced information requirements.

The purpose of this paper is to examine spatial pollutants in a policy context where a specific environmental quality goal is being sought by some regional authority. An important characteristic of the pollutant being considered is that it is also persistent. That is, the pollutant possesses qualities of a stock "resource" in that it does not decay (and is not assimilated) in the present planning period. Thus, the regional authority must consider the impact that its decisions have on the environmental quality of future time periods as well as the present [13, 16]. Proper consideration of pollutant movement and decay processes suggests that spatial and persistent pollutants are the rule rather than the exception. In fact, these two properties should logically go hand in hand. Since nearly all pollutants are spatial

[1]Technical Article No. 21101 of the Texas Agricultural Experiment Station. The author expresses his appreciation for constructive comments provided by the anonymous reviewer.

0095-0696/87 $3.00

and persistent (depending on the scope of regional concerns), formulations which ignore these properties are deficient, and the implications of these deficiencies must be explored.

Several articles dealing with pollution control dynamics have been published, but the focus is generally upon questions of economic growth and capital formation. While most consider pollution as a stock [2, 3, 8, 11, 12, 15], decay in pollution is assumed proportional to the stock, and a single utility (social welfare) function of consumption and pollution is maximized. The objective function is typically linearly separable in its arguments, and society's resources can be devoted to any combination of production activity and pollution control. The spatial properties of pollution are not considered.

The approach undertaken here is different. The relative efficacies of price-guided and quantity-guided policies are explicitly considered. Capital investment is ignored because of the extensive early literature. An extremely general view of pollution accumulation and assimilation is modelled, and spatial attributes are incorporated for additional generality. Because the possibilities of obtaining utility or social welfare functions are remote, a more realistic objective is adopted.

I. THE MODEL

In a region divided into K geographic sectors (zones, river segments, airsheds, communities, or households), let there be J firms discharging pollutant z. These firms have differing productive abilities which are specified by the implicit production functions $f^j(\mathbf{y}^j, z^j) \leq 0$ (for all j), where $\mathbf{y}^j$ is the netput vector $(N \times 1)$ of firm J's priced inputs $(-)$ and outputs $(+)$. Let $\mathbf{p}$ denote the price vector $(1 \times N)$, where all prices are nonnegative.

The social goal driving this economic model is the now familiar Baumol and Oates criterion of meeting an environmental constraint at least cost to society. While this objective does not guarantee the achievement of Pareto optimality, it is commendable in that it obviates the need for estimating abatement benefits (which is often a very costly endeavor). Here, each sector of the spatial division is assumed to be confronted with a permanent environmental standard, A^k. For generality, these standards are allowed to vary between zones, and A^k may relate to a maximum level of discharge, a maximum pollutant concentration, or a limit on any environmental index.

For each sector k, it is assumed that there exists a function which relates changes in the environmental quality of the sector to current emissions and the environmental quality experienced by neighboring sectors. Because every pollutant is persistent to some degree (that is, each unit of a pollutant will exist over some positive time interval), slow media (especially groundwater) or the choice of sufficiently small time increments will mean that pollutants discharged during any period will also be present in the region in forthcoming periods. Therefore, spatial transport functions for persistent pollutants will be necessarily dynamic. Allowing these functions to differ among sectors, we have

$$\Delta a^k(t+1) = h^k(\mathbf{a}(t), \mathbf{z}(t), w(t)) \qquad \text{for all } k,$$

where $\Delta a^k(t)$ is the change in the value of the environmental index for sector k in time t, $h^k(\cdots)$ is one of K dynamic transport functions, $\mathbf{a}(t)$ is the state of environmental quality at time t $(K \times 1)$, and $\mathbf{z}(t)$ is the vector $(J \times 1)$ of emissions during t. The scalar, $w(t)$, is included to represent the effect of dynamic factors, say weather, which are exogeneously determined; this parameter could be expanded to greater dimensions with no change in this analysis. The transport functions are quite general and capture the fact that pollution may enter a particular sector by either (a) being transported from nearby sectors or (b) being directly emitted into the sector. Natural processes of pollutant decay or assimilation can usually be accommodated or, at least, approximated within this formulation.

In this way, persistent emissions during any single time period can be shown to contribute to decreased environmental capacity (as defined by the limits A^k) in the sector in which they are emitted now and possibly in the future or in other sectors during the future. Thus, the decision-making framework must recognize both the present and future opportunity costs for the emissions of each firm.

Let the social problem of choosing optimal production programs be limited to periods $t = 0, 1, \ldots, T$ and assume that the specified production functions are time invariant. Assuming prices are stable and the discount rate is r, the regional economic problem is as follows:

$$\operatorname*{Max}_{\mathbf{Y},\mathbf{z}} \Pi = \sum_{t=0}^{T} (1+r)^{-t} \sum_j \mathbf{p}\mathbf{y}^j(t) \tag{1a}$$

subject to

$$f^j\big(\mathbf{y}^j(t), z^j(t)\big) \leq 0 \qquad \text{for all } j; \tag{1b}$$

$$\Delta a^k(t+1) = h^k(\mathbf{a}(t), \mathbf{z}(t), w(t)) \qquad \text{for all } k; \tag{1c}$$

and

$$a^k(t) \leq A^k \qquad \text{for all } k. \tag{1d}$$

To this problem we must also add the initial conditions

$$a^k(0) = a_0^k \qquad \text{for all } k. \tag{1e}$$

Thus, the problem is one of determining time paths for each $\mathbf{y}^j$ and each z^j so that the present value of the time stream of regional profits can be maximized subject to technological and environmental constraints as well as initial conditions. Since the problem can be revised and resolved during future periods, primary consideration rests in the determination of $\mathbf{y}^{j*}(0)$ and $z^{j*}(0)$ for each firm. If $z^{j*}(0)$ can be computed, then these values can be promulgated as discharge regulations. Otherwise, decentralized opportunities (e.g., economic incentives, transferable permits) for inducing firms to determine and adopt these solutions should be investigated.

This problem can be more easily managed by solving its continuous time analog. Formulation of this problem is straightforward.

$$\operatorname*{Max}_{\mathbf{Y},\mathbf{z}} \Pi = \int_0^T e^{-rt} \sum_j \mathbf{p}\mathbf{y}^j(t)\, dt \tag{2a}$$

subject to

$$f^j(\mathbf{y}^j(t), z^j(t)) \leq 0 \qquad \text{for all } j; \tag{2b}$$

$$da^k(t)/dt = \dot{a}^k(t) = h^k(\mathbf{a}(t), \mathbf{z}(t), w(t)) \qquad \text{for all } k; \tag{2c}$$

$$a^k(t) \leq A^k \qquad \text{for all } k; \tag{2d}$$

and

$$a^k(0) = a_0^k \qquad \text{for all } k; \tag{2e}$$

where

$$t \in [0, T]; \quad j = 1, \ldots J; \quad k = 1, \ldots, K;$$
$$\mathbf{y}^k \text{ is } N \times 1; \quad \mathbf{z} \text{ is } J \times 1; \quad \text{and } \mathbf{a} \text{ is } K \times 1.$$

II. THE SOCIAL OPTIMUM

Problem (2) is an optimal control problem with control variables y_n^j and z^j ($J \cdot (N + 1)$ variables) and state variables a^k. Because all constraints apply throughout the planning period, constraints can be adjoined to the objective function within the integral in the usual Lagrangian fashion. Thus,

$$L = \int_0^T \Big[e^{-rt} \sum_j \mathbf{p}\mathbf{y}^j(t) + \sum_k \lambda^k(t)\big(h^k(\mathbf{a}(t), \mathbf{z}(t), w(t)) - \dot{a}^k(t)\big)$$
$$- \sum_j \alpha^j(t) f^j(\mathbf{y}^j(t), z^j(t)) - \sum_k \mu^k(t)\big(a^k(t) - A^k\big) \Big] dt \tag{3}$$

where $\lambda^k(t)$, $\alpha^j(t)$, and $\mu^k(t)$ are the adjoint (or costate) variables of the control problem.

The Hamiltonian function for (3) encompasses all terms except those containing $\dot{a}^k$. Therefore,

$$H = \int_0^T \Big[e^{-rt} \sum_j \mathbf{p}\mathbf{y}^j(t) + \sum_k \lambda^k(t) h^k(\mathbf{a}(t), \mathbf{z}(t), w(t))$$
$$- \sum_j \alpha^j(t) f^j(\mathbf{y}^j(t), z^j(t)) - \sum_k \mu^k(t)\big(a^k(t) - A^k\big) \Big] dt \tag{4}$$

and

$$L = \int_0^T \Big[H - \sum_k \lambda^k(t) \dot{a}^k(t) \Big] dt. \tag{5}$$

To derive necessary conditions the latter terms are integrated by parts and the results substituted into (5) to obtain

$$L = \int_0^T \Big[H + \sum_k \dot{\lambda}^k(t) a^k(t) \Big] dt - \sum_k \big(\lambda_T^k a_T^k - \lambda_0^k a_0^k \big).$$

Taking the total derivative,

$$dL = \int_0^T \left[\sum_j \left(D_y^j(H) d\mathbf{y}^j + \left(\partial H / \partial z^j \right) dz^j \right) + \sum_k \left(\partial H / \partial a^k + \dot{\lambda}^k(t) \right) da^k \right] dt = 0, \tag{6}$$

where $D_y^j(H)$ denotes the $1 \times N$ vector of partial derivatives of H.

From (6) we have that optimal trajectories for control, state, and adjoint variables must satisfy

$$\partial H / \partial y_n^j = 0 \qquad \text{for all } j, n;$$

$$\partial H / \partial z^j = 0 \qquad \text{for all } j;$$

and

$$\partial H / \partial a^k = -\dot{\lambda}^k(t) \qquad \text{for all } k;$$

as well as constraints (2b), (2c), Kuhn–Tucker type conditions for the environmental constraints (2d), and initial conditions (2e). Applying these conditions to the Hamiltonian defined above, equations (7a)–(7h) identify all first-order conditions:

$$e^{-rt} p_n - \alpha^j(t) \left(\partial f^j / \partial y_n^j \right) = 0 \qquad \text{for all } j, n; \tag{7a}$$

$$\sum_k \lambda^k(t) \left(\partial h^k / \partial z^j \right) - \alpha^j(t) \left(\partial f / \partial z^j \right) = 0 \qquad \text{for all } j; \tag{7b}$$

$$\sum_\ell \left[\lambda^\ell(t) \left(\partial h^\ell / \partial a^k \right) \right] + \mu^k(t) = \dot{\lambda}^k(t) \qquad \text{for all } k; \tag{7c}$$

$$f^j\left(\mathbf{y}^j(t), z^j(t)\right) \leq 0 \qquad \text{for all } j; \tag{7d}$$

$$\dot{a}^k(t) = h^k(\mathbf{a}(t), \mathbf{z}(t), w(t)) \qquad \text{for all } k; \tag{7e}$$

$$\mu^k(t) \left(a^k(t) - A^k \right) = 0 \qquad \text{for all } k; \tag{7f}$$

$$\mu^k(t) \geq 0 \qquad \text{for all } k; \tag{7g}$$

and

$$a^k(0) = a_0^k \qquad \text{for all } k. \tag{7h}$$

Equation (7a) is nothing more than the familiar marginal conditions for productive efficiency. Equation (7b) states that, for each firm, the marginal costs (across all sectors) imposed by persistent discharges must be balanced against marginal benefits of these emissions. In this framework the benefits of an additional unit of pollution lie in an increased level of output and/or decreased inputs for the emitting firm *at the instant of discharge*. The costs of this emission are possibly borne by other firms at the same instant or all firms during the future. Because of persistence, greater discharge levels during one period may require decreased discharge during the future if environmental standards are to be maintained. Equation (7b) specifies the optimal choice of control variables to make this tradeoff.

Interpretation of (7c) is assisted by the following relationship:

$$\lambda^k(t) = \partial \Pi^*(t) / \partial a_t^k,$$

where $\Pi^*(t)$ is the maximum present value of regional profits from t to T [5, p. 352]. Thus, each λ^k is the amount of change in the objective function which can be obtained through a marginal change in the degree of pollution present in a particular area. Because increases in the state variables (the a^k's) should lead to decreases in future profitability, $\lambda^k(t)$ is nonpositive for all k. The dual variables $\alpha^j(t)$ and $\mu^k(t)$ are easily interpreted as typical Lagrange multipliers. At any instant $\alpha^j(t)$ represents the marginal value of technological advance in production for firm j, and $\mu^k(t)$ is the instantaneous value of relaxing the environmental standard A^k.

The first term of Eq. (7c) is the sum, across all sectors, of the rates of contribution by marginal increases in sector k's pollution level to pollution in all sectors of the region (where these rates are weighted by the implicit values of stock pollution quantities). In any given sector this total value (which is negative) plus the implicit value of the environmental constraint for the sector (positive) must equal the rate of change in the marginal value of the existing stock of pollution. Thus, $\dot{\lambda}^k(t)$ may be positive or negative.

Equations (7d), (7e), (7h) merely return constraints and initial conditions for problem (2). Equations (7f), (7g) are familiar Kuhn–Tucker conditions for an inequality constraint; only here a constraint on state variables is involved. These latter two equations require that if a particular region's environmental constraint is not binding at a certain instant, then the associated costate variable, $\mu^k(t)$, must be zero at that time.

III. ECONOMIC INCENTIVES

So that each firm can be induced to adopt socially optimal production strategies (as defined by Eq. (7)), it is initially presumed that each firm is subject to an economic incentive attached to discharges. In each successive period the incentive is to be revised, so there is no real need for considering control variables beyond $t = 0$. Because of the spatial specification of the problem, efficient effluent incentives will differ among firms and shall be denoted by s^j. It can be shown, however, that firms in the same zone will face the same economic incentive. For generality, it shall be assumed that there is an arbitrary and fixed quantity, $\bar{z}^j$, defining the extent to which each firm's incentive is a charge or a subsidy.[2] Discharges in excess of $\bar{z}^j$ are charged at the rate s^j, and each unit by which discharges fall below $\bar{z}^j$ are subsidized at the same rate.[3] Except for induced entry and exit by firms, aggregate discharges will be independent of the $\bar{z}^j$'s as long as these latter quantities are independent of actual discharges.

Under these conditions, the profit maximization problem faced by each firm is summarized by Eq. (8). This Lagrangian indicates that profits (inclusive of the net effluent incentive) are to be maximized subject to the production constraint.

$$L^j(\mathbf{y}^j, z^j, \delta^j) = \mathbf{p}\mathbf{y}^j + s^j(\bar{z}^j - z^j) - \delta^j f^j(\mathbf{y}^j, z^j). \tag{8}$$

[2]Alternatively, $\bar{z}^j$ can be thought of as firm j's initial endowment of transferable emission permits. In this case, s^j must be interpreted as the instantaneous rental price for permits in the jth firm's zone.

[3]Choosing $\bar{z}^j = 0$ for all j corresponds to a pure charge. Choosing $\bar{z}^j$ equal to each firm's previous discharges corresponds to a pure subsidy but is not recommendable because of opportunities for strategic behavior.

Because the firm is presumably unconcerned for the persistence of pollutant z, this problem is inherently static. Proper consideration for this persistence can only be achieved by an appropriate choice of s^j by the regional authority.

Necessary conditions for profit maximization are given by

$$p_n - \delta^j(\partial f^j/\partial y_n^j) = 0 \qquad \text{for all } n,$$

and

$$s^j + \delta^j(\partial f^j/\partial z^j) = 0.$$

Comparing these equations to (7a) and (7b), we have the requirements necessary for the social optimality of profit maximization in $t = 0$:

$$\delta^j = \alpha_0^j, \tag{9a}$$

and

$$s^j = -\sum_k \lambda_0^k(\partial h^k/\partial z^j). \tag{9b}$$

Barring production externalities, Eq. (9a) will be satisfied because private and social evaluations of technological capability will be equivalent (assuming (9b) is also satisfied).

Equation (9b) specifies the optimal level of each firm's economic incentive. Because all λ_0^k will be nonpositive and all partial derivatives of the dynamic transport equations will be nonnegative, the incentive must be positive, as expected. Therefore, the least-cost incentive for each firm is the firm's marginal rate of contributions to each sector's environmental degradation, summed across all sectors weighted by the marginal value of each sector's present stock of pollution. Since the λ^k's measure the sensitivity of regional profit to the pollution that is present in specific sectors, this incentive will provide greater abatement inducements for those firms discharging into critical areas. Because it is reasonable to presume that the dynamic pollution transport functions do not differentiate between firms discharging into the same sector, these firms should face the same economic incentives.

IV. REGULATION

Optimal regulations can also be established, and, as noted by Suchanek in the static case [18], optimal regulations and optimal incentives are related as duals. Specifically, if the system of equations (7a)–(7h) can be obtained and solved, then the optimal trajectories of firm pollutant emissions, the $z^j(t)$'s, can be promulgated as regulations. Thus, both regulations and incentives can be theoretically optimal policies in the sense that the pollution externality is resolved.

In the case of nonspatial and nonpersistent pollutants, optimal regulations will be firm specific, but the optimal incentive will be identical for all firms. This fact together with the availability of Baumol and Oates' scheme of "charge and standards" implies that incentives will be informationally superior to a system of direct controls. In that case, the informational advantage of the price-guided policy stems from the opportunity to achieve an optimal incentive without actually solving the social optimization problem. The optimal regulatory program cannot, however, be determined without explicitly specifying and solving this problem. It has been previously demonstrated that the informational superiority of incentives breaks down when the very real consideration of spatial variability in pollution loadings is introduced [9, 19, 21]. Using the analytical structure developed above, it is a relatively simple matter to obtain this same conclusion when the pollutant is persistent but not spatial.

V. THE ONE-ZONE CASE

In order to emphasize the influence of persistence on the choice between alternative price- and quantity-guided policies, assume that spatial dimensions of problem (2) have vanished and revise the solution accordingly. Equations (7a)–(7h) now become

$$e^{-rt}p_n - \alpha^j(t)(\partial f^j/\partial y_n^j) = 0 \qquad \text{for all } j, n; \tag{10a}$$

$$\lambda(t)(\partial h/\partial z^j) - \alpha^j(t)(\partial f^j/\partial z^j) = 0 \qquad \text{for all } j; \tag{10b}$$

$$\lambda(t)(\partial h/\partial a) + \mu(t) = \dot{\lambda}(t); \tag{10c}$$

$$f^j(\mathbf{y}^j(t), z^j(t)) \leq 0 \qquad \text{for all } j; \tag{10d}$$

$$\dot{a}(t) = h(a(t), \mathbf{z}(t), w(t)); \tag{10e}$$

$$\mu(t)(a(t) - A) = 0; \tag{10f}$$

$$\mu(t) \geq 0; \tag{10g}$$

and

$$a(0) = a_0. \tag{10h}$$

In addition to Eqs. (10a)–(10h), we have that the effluent of all firms affects environmental quality equally.

Profit maximization by firms facing an economic incentive is unaltered. Therefore, the optimal instantaneous one-zone economic incentive is as follows:

$$s = -\lambda_0(\partial h/\partial z). \tag{11}$$

All firms should, ideally, be subjected to the same economic incentive in this case. Furthermore, the incentive given by (11) appears to be very similar to the ordinary static economic incentive for the least-cost achievement of an environmental objective except for the information embodied in λ_0.

To see the effect of λ_0 consider the following. In the "charges and standards" procedure for a nonpersistent pollutant some best-guess incentive would be established, resulting environmental quality would be observed, and the incentive would be revised in an appropriate direction depending on whether the environmental quality objective is over- or under-achieved. Even if the time required to converge upon the optimal choice of incentive is disregarded, this procedure is unavailable for persistent pollutants because the incentive cannot be recursively improved without knowing the optimal trajectory of environmental quality, $a^*(t)$. If $a^*(t)$ is known, then the scheme is applicable, and incentives might have informational superiority over regulations. However, since it is not possible to establish $a^*(t)$ without solving (10a)–(10h), incentives do not offer an informational advantage over direct controls when real considerations of persistence are introduced. Furthermore, if $a^*(t)$ is computed, then $\lambda^*(t)$ will also be known, and the optimal incentive can be directly established via (11) rather than relying on an iterative algorithm.

Therefore, just as the spatial qualities of pollution invalidate the oft-promoted advantage of price guides over quantity guides, attention to the influence of persistence upon policy selection demonstrates no apparent grounds for holding economic incentives to be meritorious. Transferable pollution permits fall prey to the same fundamental problem. It is not possible to determine the quantity of emission permits to issue without knowing the optimal trajectory of environmental quality.[4] While economic incentive or market-oriented policies may be preferable on the basis of some notion of quasi-optimality or second-best criterion or, perhaps, some principle of equity, such policies cannot be demonstrated to be allocatively superior in this modelling framework (which is quite general). In fact, since the present analysis has been founded upon the cost-effective accomplishment of an exogeneously determined environmental target rather than a full treatment of the costs and benefits of environmental quality, we are already working within a second-best framework. Only by adopting third or fourth-best criteria, such as when spatial and dynamic dimensions of the policy problem are ignored or a steady-state situation is assumed, can the informational superiority of price-guided mechanisms be demonstrated.

VI. JUMPS IN POLICY PARAMETERS

Continuing, for now, to highlight pollution persistence by examining the one-zone case, it must be noted that the social control problem includes a single state variable inequality constraint (SVIC). In the generalized multiple sector case, Eq. (2d) represents K such constraints. Control problems with SVICs pose additional complications which may be relevant to policy selection. The inherent difficulties of SVIC control problems have been previously recognized within a pollution context [3], but the setting was quite different than that pursued here, and the potential implications for policy were not explored.

The most important characteristic of a SVIC is the absence of any control variable. However, when the SVIC is binding, the space of admissible controls is implicitly reduced. Within the one-zone case, an active SVIC ($a(t) = A$) implies that we must require

$$\dot{a}(t) \leq 0$$

or, using (2c),

$$h(a(t), \mathbf{z}(t), w(t)) \leq 0. \tag{12}$$

Thus, along a constrained arc of $a^*(t)$, the SVIC causes the control variable

[4] Note that permitted emissions must change over time in order to be optimal. Thus, permits will exhibit a time dimension in their definitions.

inequality constraint given by (12) to be in force. Off of the constrained arc the SVIC can be ignored. Since $h(\dots)$ is an explicit function of $\mathbf{z}(t)$, we have a first-order SVIC in that the first time derivative of the SVIC contains at least one control variable [17]. There are alternative methods of handling SVIC control problems (see, for example, [6, 10, 14]), and some of these methods depend on the order of the SVIC.

The most interesting feature of a solution to a SVIC control problem is that the adjoint variable associated with the bounded state variable is no longer continuous. As the environmental quality constraint becomes active in the one-zone case, $\lambda(t)$ experiences a finite, positive jump in value. Similarly, $\lambda(t)$ will also undertake a finite, positive jump at the instant that $a(t) \leq A$ becomes nonconstraining. At every juncture between constrained and unconstrained trajectories of $a^*(t)$ Eq. (12) will logically hold with strict equality, and $a^*(t)$ is therefore tangent to the constraint boundary at all such junctures. There will be a finite number of these junctures and, hence, a finite number of jumps in $\lambda(t)$.

In the procedure used to obtain necessary conditions (10) and (7), the SVICs was (were) directly adjoined to the Hamiltonian. The more traditional approach within the literature to such problems has been to adjoin a sufficient number of time derivatives to capture a control variable explicitly (e.g., [6, 10]). For the nonspatial case this requires only Eq. (12) because the SVIC is first order. Caution must be exercised when interpreting the results of these alternative procedures; the multipliers $\lambda(t)$ and $\mu(t)$ have different interpretations and properties depending on which of the two methods is adopted. For the method employed here, both of these multipliers are discontinuous at juncture points of the environmental quality trajectory [14, p. 338]. Furthermore, the following jump conditions hold at all such juncture points [14, p. 380; 7, pp. 220, 221]:

$$\lambda(\tau^-) - \lambda(\tau^+) = b \cdot \frac{\partial(A - a(t))}{\partial a(t)} = -b \tag{13}$$

and

$$b \geq 0 \tag{14}$$

where τ is a point of discontinuity and $\lambda(\tau^-)$ is the value of λ just before the jump and $\lambda(\tau^+)$ is the value just afterwards. Equations (13) and (14) imply

$$\lambda(\tau^-) < \lambda(\tau^+) \tag{15}$$

at all junctures. Thus, $\lambda(t)$ jumps positively at each instant environmental quality enters or exits a constrained arc (Fig. 1).

The policy significance of $\lambda(t)$ being discontinuous is reflected by Eq. (11), which specifies the efficient choice of economic incentive. Expanding (11) to investigate the continuity of the optimal incentive at a juncture, we have

$$s(\tau^-) - s(\tau^+) = -\lambda(\tau^-) \cdot (\partial h/\partial z|_{\tau^-}) + \lambda(\tau^+) \cdot (\partial h/\partial z|_{\tau^+}). \tag{16}$$

If the optimal incentive is continuous at all junctures with the SVIC, then the left-hand side of (16) must equal zero. If the incentive changes continuously,

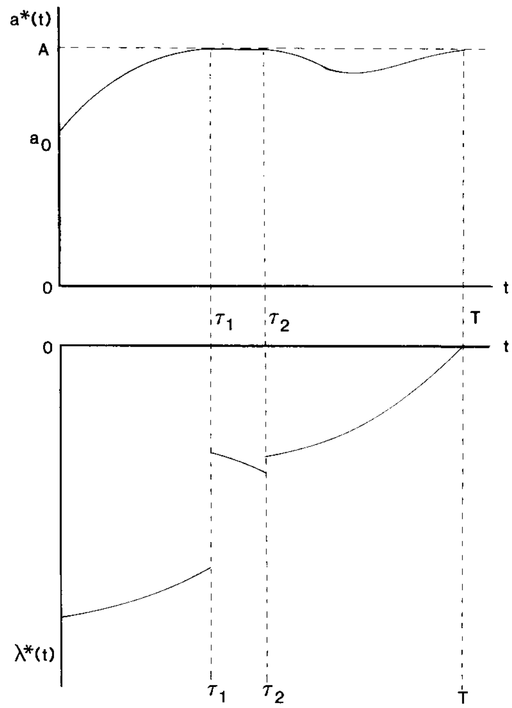

FIG. 1. Jumps in λ corresponding to a constrained arc of the environmental standard.

however, then so will the control variable z which implies that the two partial derivatives in (16) are equal. This implies

$$s(\tau^-) - s(\tau^+) = (-\lambda(\tau^-) + \lambda(\tau^+)) \cdot C,$$

where C is some constant. Applying (15), we have a contradiction because the incentive must jump at τ; thus the optimal incentive must jump at junctures with the SVIC boundary. The direction of such jumps is uncertain and depends on the nature of the dynamic transport function and the responsiveness of firms to changes in s.

In the multiple zone case, Eqs. (13) and (14) become

$$\lambda^k(\tau^-) - \lambda^k(\tau^+) = \sum_{\ell=1}^{K} b^{k\ell}.(\partial(A^\ell - a^\ell(t))/\partial a^k) = -b^{kk}$$

and

$$b^{kk} \geq 0.$$

Therefore, as the environmental constraint becomes binding (or nonbinding) in a particular sector, the adjoint variable corresponding to that constraint will experience a positive jump. From Eq. (9b) this implies that the economic incentive for this sector will be the only one to move discontinuously.

Because discrete time formulations are most apropos for policy operations, it is hard to know how much of these findings are relevant to actual policy settings. To the extent that discrete time policies seek to approximate continuous time ideals, however, discrete environmental programs involving persistent pollutants should be emulating discontinuous time paths. The practical difficulties that can arise here are uncertain, but they are clearly disconcerting. In the case of observe-and-revise schemes such as the "charges and standards" procedure, discrete jumps in efficient charges should be expected, and, given the above results (especially Eq. (16)), the direction of these jumps may be analytically determined.

VII. CONCLUSIONS

Common sense dictates that all pollutants are spatial and persistent—to varying degrees. Because of earlier literature dealing with the spatial attributes of pollution, the influence of persistence has been emphasized here, but spatial characteristics have been included for generality. The developed optimal control model is deterministic and presumes that the social objective is to obtain arbitrary environmental standards (which may vary by zone) at least cost to society. When the solution to this problem is used to identify trajectories for spatially dependent economic incentives, the traditionally acknowledged informational advantage of economic incentive and transferable permit policies over regulations breaks down. In this case, it takes just as much information to determine efficient economic incentives or permit quantities as it does to calculate efficient regulations. Therefore, price guides do not offer an advantageous, decentralized alternative to regulation. Previous literature recognizes this conclusion in the case of spatial properties only. Inspection of the one-zone (nonspatial) case demonstrates that persistence alone will also invalidate the informational edge of price guides over quantity guides.

A secondary result concerns the nature of least-cost pollution policies at the instant environmental standards become constraining or nonconstraining. The imposition of environmental constraints introduces state variable inequality constraints into the control problem. In the one-zone case optimal policy parameters (regulations or incentive levels) are likely to experience discontinuous jumps at the moment of entering or exiting a time interval when the environmental standard is binding. The same conclusion holds in the spatial case, but policy parameters in one zone do not jump discontinuously as a result of an environmental constraint in a neighboring zone, as might be expected.

The potential discontinuity of policy parameters is an outcome of the "least-cost" modelling framework. In a model where the benefits of pollution abatement can be considered explicitly, this result will not emerge. The operational ease of standard-oriented policies, however, is very compelling and implies that discontinuous jumps in policy parameters should be anticipated.

REFERENCES

1. W. J. Baumol and W. E. Oates, "The Theory of Environmental Policy," Prentice–Hall, Englewood Cliffs, N.J. (1975).
2. D. A. d'Arge and K. C. Kogiku, Economic growth and the environment, *Rev. Econ. Stud.* **40**, 61–78 (1973).

3. B. A. Forster, On a one state variable optimal control problem: Consumption-pollution trade-off, *in* "Applications of Control Theory to Economic Analysis" (J. D. Pitchford and S. J. Turnovsky, Eds.), North-Holland, New York (1977).
4. J. V. Henderson, Externalities in a spatial context, *J. Public Econ.* **7**, 89–110 (1977).
5. M. D. Intriligator, "Mathematical Optimization and Economic Theory," Prentice–Hall, Englewood Cliffs, N.J. (1971).
6. D. H. Jacobson, M. M. Lele, and J. L. Speyer, New necessary conditions of optimality for control problems with state-variable inequality constraints, *J. Math. Anal. Appl.* **35**, 255–284 (1971).
7. M. I. Kamien and N. L. Schwartz, "Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management," North-Holland, New York (1981).
8. E. Keeler, M. Spence, and R. Zeckhauser, The optimal control of pollution, *J. Econ. Theory* **4**, 19–34 (1971).
9. K. Mäler, "Environmental Economics: A Theoretical Inquiry," Johns Hopkins Press for Resources, for the Future, Baltimore (1974).
10. K. Martensson, A constraining hyperplane technique for state variable constrained optimal control problems, *J. Dynamic Systems, Meas. Cont.* **95**, 380–389 (1973).
11. C. G. Plourde, A model of waste accumulation and disposal, *Canad. J. Econ.* **5**, 119–125 (1972).
12. Y. Sakawa and Y. Hashimoto, Control of environmental pollution and economic growth: Modelling and numerical solution, *Appl. Math. Optim.* **4**, 385–400 (1978).
13. A. Scott, Some economic aspects of persistent pollutants, *in* "Workshop on Economic and Legal Enforcement Mechanisms" (P.A. Bonner, Ed.), International Joint Commission, Windsor, Ontario (1977).
14. A. Seierstad and K. Sydsaeter, Sufficient conditions in optimal control theory, *Internat. Econ. Rev.* **18**, 367–391 (1977).
15. V. L. Smith, Dynamics of waste accumulation: disposal versus recycling, *Quart. J. Econ.* **86**, 600–616 (1972).
16. V. K. Smith and J. V. Krutilla, Resource and environmental constraints to growth, *Amer. J. Agricultural Econ.* **61**, 395–408 (1979).
17. J. L. Speyer and A. E. Bryson, Jr., Optimal programming problems with a bounded state space, *Amer. Inst. Aeronautics Astronautics J.* **6**, 1488–1491 (1968).
18. G. L. Suchanek, Information, optimality and pollution control, *J. Public Econ.* **12**, 99–114 (1979).
19. T. H. Tietenberg, Derived decision rules for pollution control in a general equilibrium space economy, *J. Environ. Econom. Manage.* **1**, 3–16 (1974).
20. T. H. Tietenberg, "Emissions Trading: An Exercise in Reforming Pollution Policy," Resources for the Future, Washington, D.C. (1985).
21. T. H. Tietenberg, On taxation and the control of externalities: Comment, *Amer. Econ. Rev.* **64**, 462–465 (1974).