Managing water supply shortage
Interruption vs. pricing

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Received March 1992, final version received July 1992

Supply shortage is a common problem faced by a consumer. Solutions for allocating the limited supply include rationing, queuing, interruption and pricing. While previous research has examined the welfare losses generated by each solution, there is little evidence on their relative magnitudes within a common framework. The objective of this paper is to specify a model of consumer behaviour under service interruption to estimate the exact welfare loss of service interruption. The same model is used to estimate the loss of a price increase intended to resolve a supply shortage. Using water consumption data collected for Hong Kong, we find that relative to pricing, service interruption is inefficient for water shortage management.

1. Introduction

Supply shortage is a common problem faced by a consumer. Solutions for allocating the limited supply include rationing, queuing, interruption and pricing. Quantity rationing refers to placing a limit on the total amount of consumption. An example is the fixed number of work hours (and therefore leisure hours) per week observed by a worker in the labour market [see Deaton and Muellbauer (1981) and Kapteyn et al. (1990)]. Queuing allocates supply by imposing waiting costs on a consumer during times of price control. An example is the U.S. gasoline crisis in the 1970s [see Frech and Lee (1987) and Deacon and Sonstelie (1989)].

 Interruption is a complete disruption of supply. Good examples are public utility services. During the period of interruption, consumption of a service is zero; otherwise, a consumer may purchase an unlimited amount at the prevailing price. This differentiates service interruption from quantity rationing. For instance, random electric power outages occur because of rotating blackouts implemented by an electric utility to resolve a capacity shortage.

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*This research is funded by CPHK Small Scale Research Grant no. 903107. I thank S. Chan and D. Ng for their research assistance. Comments from two referees helped to improve this paper substantially. Without implications, all errors are mine

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SSDI 0047-2777(93)01380-S
Such power outages impose large economic costs on consumers as reported by Munasinghe et al. (1988) and Woo and Pupp (1992). Because most of the outage cost estimates are based on contingent valuation survey data, they have been criticized as implausible.¹ Hartman et al. (1990, 1991) argue that the large estimates are partially due to the status quo bias in consumer decision-making, as documented by Kahneman et al. (1991). However, cost estimates based on survey data remain controversial since they cannot be readily verified in a well-established market environment with many consumers and repeat transactions.

Another example is that a water utility may use service interruption to reduce consumption during periods of severe drought. In contrast to electricity, there is no empirical evidence on the economic costs of water service interruption, even though Riley and Scherer (1979) and Krzysztofowicz (1986) show that the information is essential to optimal water pricing and planning. To wit, Moncur (1987) estimates the effect of rationing on residential water consumption. Hamilton et al. (1989) compute the benefit of diverting irrigation water for hydro power production. Sengupta and Khalili (1986) estimate the shadow value of irrigation water shortage using quadratic programming. Whittington et al. (1990) apply the contingent valuation method to estimate the willingness to pay for the access to water service in a less developed country. Though related, these studies do not estimate the welfare loss of water service interruption.

Responsive pricing, first proposed by Vickrey (1971), is an efficient solution to resolve a shortage of utility services. In the case of electricity, Bohn et al. (1984) recommend the use of spot pricing to continuously equate demand and supply. Since the implementation of spot pricing may be costly, an alternative is forward contracts that prescribe the allocation of the limited supply during a shortage [see Chao and Wilson (1987), Wilson (1989), Spulber (1990) and Woo (1990)]. Under certain conditions, forward contracts can be as efficient as spot pricing. Both spot pricing and forward contracts welfare dominate random service interruption. In view of this finding, they have been implemented by some electric utilities in California (e.g. Pacific Gas and Electric Company and Southern California Edison) and in New York (e.g. Niagara Mohawk Power Corporation).

Parallel to the electricity pricing literature, there is general consensus supporting the use of prices to efficiently allocate scarce water resources.² However, pricing continues to play little role in water shortage management [see DWR (1987), Schuster (1987) and Schwartz (1988)]. Growing demand


for water is met by new supplies and conservation programs (e.g. public education, improved irrigation practice, leak detection and low flow shower heads). If a severe shortage develops, a water utility implements such programs as quantity rationing or service interruption to reduce water consumption.

The objectives of this paper are several. First, we specify in section 2 a model of consumer behaviour under service interruption which can be implemented using market data. Our approach differs from Deaton and Muellbauer (1981) and Kapteyn et al. (1990) who model the effect of a quantity constraint on consumer behaviour. Our model is also different from Frech and Lee (1987) and Deacon and Sonstelie (1989) who use the value of waiting time to analyse the welfare loss of rationing-by-queuing. Our model yields a rigorous measure of welfare loss of service interruption. The measure is exact in the sense of Hausman (1981). We parameterize the welfare loss resulting from service interruption or price increase using the same representation of consumer behaviour. Second, we demonstrate the fruitfulness of our approach by implementing it in section 4 using water consumption data of Hong Kong described in section 3. Since our estimates of welfare loss are based on actual market data, they are free from the common criticisms often levied on the use of survey data. Third, we compare the welfare loss of water service interruption with that of a price increase which yields the same amount of consumption reduction. This comparison shows that service interruption is very inefficient relative to pricing for water shortage management.

2. The model

Let \( S > 0 \) be the time interval during which the supply of service is available at a price equal to \( P \). \( S \) is known in advance. Let \( I \geq S \) be the entire time period in which a consumer selects his or her consumption bundle. The duration of service interruption is \((I - S) \geq 0\). The indirect utility function of a consumer with income \( Y \) is

\[
V(P, Y, S) = \max_{\{q_{i}\}} \left( \int_{0}^{S} W(q_{i}) \, dq_{i}; \, Y - P \int_{0}^{S} q_{i} \, dq_{i} \right),
\]

where \( U(\cdot) \) is the direct utility function. In eq. (1), \( W(q_{i}) \) is a sub-utility function which is increasing and concave in service consumption, \( q_{i} \). We assume that \( U(\cdot) \) is increasing and concave in \( W(q_{i}) \) and the numeraire \((Y - P \int_{0}^{S} q_{i} \, dq_{i})\); see Koenker (1979).

If \( V(P, Y, S) \) is twice differentiable in \( S \), eq. (1) implies

\[
\frac{\partial V}{\partial S} \geq 0 \quad \text{and} \quad \frac{\partial^2 V}{\partial S^2} \leq 0.
\]
Thus, the marginal utility of \( S \) is positive and diminishing. We make additional assumptions regarding the effect of \( S \) on \( \partial V / \partial P \) and \( \partial V / \partial Y \):

**Assumption 1.** \( \partial^2 V / \partial P \partial S \leq 0 \), implying that an increase in \( P \) reduces the marginal utility of \( S \).

**Assumption 2.** \( \partial^2 V / \partial Y \partial S \geq 0 \), implying that an increase in \( S \) increases the marginal utility of income.

Invoking Roy's Identity yields

\[
-(\partial V / \partial P) / (\partial V / \partial Y) = \int_0^S q_i(P, Y, S) \, ds = Q(P, Y, S),
\]

the observed total consumption. If the service is a normal good, then consistency with consumer maximization requires \( Q(P, Y, S) \) to be decreasing in \( P \) and increasing in \( Y \). We further assume:

**Assumption 3.** \( \partial Q / \partial S = \left[ -(\partial V / \partial Y)(\partial^2 V / \partial S \partial P) \right]^{(+)} \)

\[+\left(\partial V / \partial P\right)(\partial^2 V / \partial Y \partial S)/(\partial V / \partial Y)^2 \geq 0. \]

This assumption is imposed to recognize that total use increases with supply availability as measured by \( S \).

We shall use eqs. (2)–(4) and Assumptions 1–3 to specify an empirical version of \( V(P, Y, S) \) that is consistent with consumer maximization.

Using \( V(P, Y, S) \), we define the exact welfare loss of service interruption when the supply duration is reduced from \( S_0 \) to \( S_1 \). Extending Hausman (1981), this loss is the Hicksian compensating variation \( (CV_S) \) for service interruption implicitly measured by

\[
V(P, Y + CV_S, S_1) = V(P, Y, S_0). \tag{5}
\]

\( CV_S \) in eq. (5) is perfectly general. If \( S_0 = I \) and \( S_1 < I \), then \( CV_S \) represents the exact welfare measure of service interruption with duration \( (S_0 - S_1) \). If \( I > S_0 > S_1 \), \( CV_S \) measures the incremental welfare loss of an increase in the interruption duration. For \( CV_S \) to be able to rank alternative supply regimes meaningfully, the following conditions hold:

**C.1.** \( CV_S(P, Y, S_0, S_0) = 0 \).

**C.2.** \( CV_S(P, Y, S_0, S_1) > 0 \).
C.3. $CV_S(P, Y, S_0, S_1) > CV_S(P, Y, S_0, S'_1)$ if and only if $S_1 < S'_1$.

To compare the welfare loss of service interruption with that of a price increase, we use the concept of virtual price [see Tobin and Houthakker (1951)]. Let $VP$ be the virtual price so that $Q(VP, Y, S_0) = Q(P, Y, S_1)$. In other words, $VP$ is the 'imagined' price that rationalizes the observed consumption $Q(P, Y, S_1)$. Assumption 3 (i.e. $\partial Q/\partial S \geq 0$) and the fact that $Q(P, Y, S)$ is decreasing in $P$ imply $VP \geq P$ whenever $S_0 \geq S_1$. We can now define the exact welfare measure for a price increase that has the same effect on consumption as $(S_0 - S_1)$. This measure is $CV_P$ implicitly measured by

$$V(VP, Y + CV_P, S_0) = V(P, Y, S_0).$$

If $CV_P < CV_S$, the pricing strategy is said to be more efficient than the interruption strategy.

For empirical implementation, we consider two functional forms for $Q(P, Y, S)$ which in turn determine the parametric specifications of $V(P, Y, S)$, $CV_S$ and $CV_P$. They are the double-log and linear specifications.

Several reasons support our interest in the double-log and linear forms. First, these forms have been used extensively in prior studies on price responsiveness of water demand [see, for example, Agthe and Billings (1980), Agthe et al. (1986) and Deller et al. (1986)]. Thus, our estimates of price and income elasticities can be readily compared with previous findings. Second, Hausman (1981) shows that these empirically popular and easy-to-implement functional forms are consistent with utility maximization and they can be used to derive exact welfare loss measurements. Third, the linear form is unrestrictive in that it allows the elasticity estimates to vary with quantity demanded. Finally, the indirect utility function for a demand function with higher order terms (e.g. quadratic or translog) is complicated; and as a result, welfare loss calculations become difficult to implement.

Under the double-log specification,

$$Q(P, Y, S) = AP^\alpha Y^\beta S^{-(1 - \phi)}.$$  

$Q(P, Y, S)$ is well behaved if $A > 0$, $\alpha < 0$ and $\beta > 0$. Assumption 3 requires $(1 - \phi) \geq 0$. Corresponding to the double-log consumption function is the following indirect utility function:

$$V(P, Y, S) = -ASP^{1 + \alpha} / (1 + \alpha) + Y^{1 - \beta} S^\phi / (1 - \beta).$$

The monthly water expenditure is a very small fraction (almost zero) of the monthly income, thus posting a difficulty in the estimation process to ensure that the coefficient estimates satisfy the regularity conditions (e.g. positive expenditure share for all observations in the sample) for a valid second-order approximation. For a discussion on the global properties of various flexible forms, see Barnett and Lee (1985).
If \( \alpha = -1 \) (or \( \beta = 1 \)), we replace the price (or income) term in \( V(P, Y, S) \) by \( \ln P \) (or \( \ln Y \)). From eq. (2), \( \partial V/\partial S \geq 0 \) requires \( \phi \geq 0 \) and \( \partial^2 V/\partial S^2 \leq 0 \) requires \( \phi \leq 1 \). Thus \( 1 \geq \phi \geq 0 \). \( V(P, Y, S) \) satisfies Assumptions 1 and 2.

Using eq. (5) we find

\[
CV_d(P, Y, S_0, S_1) = \left\{ [1 - \beta] (S_1 - S_0)/(1 - \alpha) S_1 Y^\beta \right\} P Q(P, Y, S_1)
\]

\[
+ (S_0/S_1)^\beta Y^{1-\beta} \right\}^{1/(1-\beta)} - Y.
\]  

Eq. (7) suggests that the double-log model yields a welfare loss measurement that depends on \( P, Y, S_0 \) and \( S_1 \). Condition C.1 is met as \( CV = 0 \) when \( S_0 = S_1 \). Since the terms in curly brackets on the right-hand side of eq. (7) have opposite signs, we need to verify conditions C.2 and C.3 empirically.

For the pricing strategy, we use eq. (6) and Hausman (1981) to find

\[
CV_p(P, VP, Y, S_0) = \left\{ [(1 - \beta)/(1 + \alpha)]\left[ VPQ(VP, Y, S_0)
\right.ight.

\[ - P Q(P, Y, S_0) \right\] + Y^{1-\beta} \right\}^{1/(1-\beta)} - Y.
\]  

A comparison between eqs. (7) and (8) reveals that \( CV_d(\cdot) \) and \( CV_p(\cdot) \) are very different. This difference allows us to compare the relative efficiency of the two strategies intended for consumption reduction.

Under the linear specification,

\[
Q(P, Y, S) = A + \alpha P + \beta Y + (1 - \phi) S.
\]

For \( Q(P, Y, S) \) to be well behaved, \( \alpha < 0 \) and \( \beta > 0 \). Assumption 3 requires \( (1 - \phi) \geq 0 \). The corresponding indirect utility function is

\[
V(P, Y, S) = e^{-\beta P}[Y + 1/\beta(A + \alpha/\beta + \alpha P + (1 - \phi) S)].
\]

\( V(P, Y, S) \) satisfies eq. (2) and Assumptions 1 and 2. Using eq. (5), we find

\[
CV_d(P, Y, S_0, S_1) = (1 - \phi) (S_0 - S_1)/\beta.
\]  

Eq. (9) indicates that \( CV_d(\cdot) \), based on a linear demand function, is proportional to \( (S_0 - S_1) \) but is independent of \( P \) and \( Y \). Moreover, \( CV_d \) meets conditions C.1–C.3.

We use eq. (6) and Hausman (1981) to find

\[
CV_p(P, VP, Y, S_0) = (1/\beta) \left\{ e^{\beta\left[ Q(P, Y, S_0) + \alpha/\beta \right]}
\right.

\[ - [Q(P, Y, S_0) + \alpha/\beta] \right\}.
\]
Table 1

Hong Kong water service interruption history for the period 1973–1990.

<table>
<thead>
<tr>
<th>Event number</th>
<th>Starting date</th>
<th>Ending date</th>
<th>Duration (days)</th>
<th>Daily unserved hours</th>
<th>Time-of-day</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25 September 1974</td>
<td>8 October 1974</td>
<td>14</td>
<td>8</td>
<td>10.00 p.m.–6.00 a.m.</td>
</tr>
<tr>
<td></td>
<td>9 October 1974</td>
<td>17 October 1974</td>
<td>9</td>
<td>14</td>
<td>11.00 a.m.–4.00 p.m. and 9.00 p.m.–6.00 a.m.</td>
</tr>
<tr>
<td>2</td>
<td>1 June 1977</td>
<td>4 July 1977</td>
<td>34</td>
<td>8</td>
<td>10.00 p.m.–6.00 a.m.</td>
</tr>
<tr>
<td></td>
<td>5 July 1977</td>
<td>18 April 1978</td>
<td>288</td>
<td>14</td>
<td>11.00 a.m.–4.00 p.m. and 9.00 p.m.–6.00 a.m.</td>
</tr>
<tr>
<td>3</td>
<td>8 October 1981</td>
<td>25 October 1981</td>
<td>18</td>
<td>8</td>
<td>10.00 p.m.–6.00 a.m.</td>
</tr>
<tr>
<td></td>
<td>26 October 1981</td>
<td>4 May 1982</td>
<td>191</td>
<td>14</td>
<td>11.00 a.m.–4.00 p.m. and 9.00 p.m.–6.00 a.m.</td>
</tr>
<tr>
<td></td>
<td>5 May 1982</td>
<td>28 May 1982</td>
<td>24</td>
<td>8</td>
<td>10.00 p.m.–6.00 a.m.</td>
</tr>
</tbody>
</table>

Source: Hong Kong Monthly Digest of Statistics, various years.

Similar to the case of the double-log, $CV_q(\cdot)$ is very different from $CV_p(\cdot)$. Calculating welfare losses using eqs. (7)–(10) requires estimating $Q(P, Y, S)$ using actual market data to be described below.

3. Data

Precise estimation of $Q(P, Y, S)$ requires data with sufficient variations in $(Q, P, Y, S)$.$^4$ Such data are available for per capita water use in Hong Kong. Table 1 describes the water service interruption history for the period 1973–1990. Because of severe drought, the Hong Kong Water Supplies Department (HKWSD) used service interruption three times to reduce demand. The number of days with service interruption ranged from 23 to 322. Each interruption event consisted of two stages. The first stage involved eight unserved hours per day. When the supply shortage worsened, the HKWSD implemented the second stage by increasing the number of unserved hours to 14 per day.

These interruptions were implemented after turning off the supply to public swimming pools, soccer fields, parks and fountains. The interruptions were highly publicized prior to their implementation, and consumers were well informed so as to take actions to mitigate the interruption effects (e.g.

$^4$For the case of electricity, disaggregated data on $(Q, P, Y)$ are readily available from the billing records of an electric utility. Although generation outages are rare, the supply duration per month by customer location can be constructed from the utility's records of power outages due to distribution network failures [see Hartman et al. (1990, 1991)].
purchase of water buckets). Consumption reduction was accomplished by complete service disruption with a duration ranging from eight to fourteen hours per day. During the unserved hours, water service to all residential and commercial buildings were shut off by manually closing the valves in the streets in Hong Kong. However, service continued for clinics and hospitals, fire and police stations, power plants, large hotels and industrial firms. In contrast, water rationing in Hawaii described by Moncur (1987) is a quantity constraint which a user can violate by paying a fine.\(^5\)

Table 2 describes the monthly data for estimating \(Q(P, Y, S)\).\(^6\) We use the aggregate data because of the lack of information on consumption by rate class (residential, commercial and others). Moreover, accurate and precise estimation of \(Q(P, Y, S)\) requires subtracting from Hong Kong's total use the aggregate consumption of the water users unaffected by the interruptions. However, such detailed information is unavailable from the HKWSD. For empirical implementation, we assume that the 'correctly' measured but unobserved consumption is proportional to the observed consumption, resulting in a possible measurement error to be captured by the random disturbance term of the demand equation; see eq. (11) below.

For the last year (1980) that the HKWSD published the annual sales by rate class, residential use contributed 36.2 percent of total water consumption in Hong Kong. Historic rate schedules indicate that residential use was billed under inverted block rates while non-residential use was subject to a flat \$\!/m^3\) charge. Thus, the average price of water use may be endogenous, an issue to be resolved in the estimation process.\(^7\)

4. Results

Without any prior knowledge about the specific form for \(Q(P, Y, S)\), we begin our analysis by estimating a Box–Cox monthly consumption function:

\[
Q(\lambda) = \text{Intercept} + \alpha P(\lambda) + \beta Y(\lambda) + (1 - \phi) S(\lambda) \\
+ \sum_j a_j W_j + \sum_k b_k D_{kl} + c N(\lambda) + u, 
\]

\((11)\)

\(^5\)Thus, this is not really a case of quantity rationing. Instead, one may view it as a multi-block rate schedule with a large marginal price for consumption above the quantity constraint. See Hausman (1985) for the econometrics of non-linear budget sets.

\(^6\)We choose April 1973–March 1984 to be our sample period for several reasons. First, there was no service interruption after 1982 because of increasing imports from China. The share of Hong Kong's aggregate consumption met by Chinese imports in 1984 was approximately 0.4 and rose to 0.6 in 1990. Second, the billing frequency was changed in April 1984 from once a month to once every three months. This change in billing policy may complicate a customer's understanding of the water bill. Finally, there was substantial economic growth in the mid-1980s which may cause a structural change in the per capita use of water. Further details on data construction are available in the appendix.

\(^7\)For a thorough discussion on this issue, see Agthe et al. (1986) and Deller et al. (1986).
Descriptive statistics of monthly data for estimating per capita water use in Hong Kong sample period: April 1973–March 1984 (132 observations) prices and income in constant HK$ (Consumer price index for April 1973 = 1.0).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_t$</td>
<td>Monthly per capita water use (cubic meter or m$^3$)</td>
<td>5.374</td>
<td>10.366</td>
<td>7.726</td>
<td>1.043</td>
</tr>
<tr>
<td>$P_t$</td>
<td>Monthly average rate ($/m^3$)</td>
<td>0.308</td>
<td>0.478</td>
<td>0.377</td>
<td>0.034</td>
</tr>
<tr>
<td>$Y_t$</td>
<td>Monthly per capita income ($)</td>
<td>661.921</td>
<td>1,336.39</td>
<td>1,007.64</td>
<td>215.310</td>
</tr>
<tr>
<td>$S_t$</td>
<td>Monthly supply hours</td>
<td>280.000</td>
<td>744.000</td>
<td>673.330</td>
<td>140.740</td>
</tr>
<tr>
<td>$W_{it}$</td>
<td>(Rainfall – evaporation): actual – normal (mm/month)</td>
<td>$\bar{W}_{it}$</td>
<td>$\bar{W}_{it}$</td>
<td>$\bar{W}_{it}$</td>
<td>$\bar{W}_{it}$</td>
</tr>
<tr>
<td>$W_{it}$</td>
<td>Average temperature: actual – normal (°C)</td>
<td>$-3.200$</td>
<td>$3.000$</td>
<td>$0.053$</td>
<td>$1.015$</td>
</tr>
<tr>
<td>$D_{it}$</td>
<td>= 1, if first quarter; = 0, otherwise</td>
<td>$0.000$</td>
<td>$1.000$</td>
<td>$0.250$</td>
<td>$0.435$</td>
</tr>
<tr>
<td>$D_{it}$</td>
<td>= 1, if second quarter; = 0, otherwise</td>
<td>$0.000$</td>
<td>$1.000$</td>
<td>$0.250$</td>
<td>$0.435$</td>
</tr>
<tr>
<td>$D_{it}$</td>
<td>= 1, if third quarter; = 0, otherwise</td>
<td>$0.000$</td>
<td>$1.000$</td>
<td>$0.250$</td>
<td>$0.435$</td>
</tr>
<tr>
<td>$N_t$</td>
<td>Number of calendar days per month</td>
<td>$28.000$</td>
<td>$31.000$</td>
<td>$30.440$</td>
<td>$0.813$</td>
</tr>
<tr>
<td>$Z_{it}$</td>
<td>Monthly last residential block rate ($/m^3$)</td>
<td>$0.517$</td>
<td>$1.657$</td>
<td>$1.072$</td>
<td>$0.313$</td>
</tr>
<tr>
<td>$Z_{it}$</td>
<td>Monthly first residential block rate ($/m^3$)</td>
<td>$0.000$</td>
<td>$0.152$</td>
<td>$0.050$</td>
<td>$0.067$</td>
</tr>
<tr>
<td>$Z_{at}$</td>
<td>Average of block rates ($/m^3$)</td>
<td>$0.258$</td>
<td>$0.677$</td>
<td>$0.504$</td>
<td>$0.113$</td>
</tr>
<tr>
<td>$Z_{at}$</td>
<td>$Z_{at} \times Z_{at}$ less bill for $Z_{at}$ at actual rates</td>
<td>$4.704$</td>
<td>$26.908$</td>
<td>$15.793$</td>
<td>$6.594$</td>
</tr>
<tr>
<td>$Z_{at}$</td>
<td>Number of blocks (residential)</td>
<td>$2.000$</td>
<td>$3.000$</td>
<td>$3.364$</td>
<td>$1.499$</td>
</tr>
<tr>
<td>$Z_{at}$</td>
<td>Sum of block quantities (residential) (m$^3$)</td>
<td>$9.100$</td>
<td>$22.700$</td>
<td>$17.791$</td>
<td>$5.104$</td>
</tr>
<tr>
<td>$Z_{ct}$</td>
<td>Monthly commercial water rate ($/m^3$)</td>
<td>$0.523$</td>
<td>$0.880$</td>
<td>$0.654$</td>
<td>$0.069$</td>
</tr>
</tbody>
</table>

where $X_t(\lambda) = (X_t^\lambda - 1)/\lambda$, a Box–Cox function with parameter $\lambda$ for $X_t = Q_t, P_t, Y_t, N_t$; and $W_{it}, D_{it}$ and $N_t$ are conditioning variables defined in table 2 to control for their respective effects on $Q_t$. Since the data are monthly series, we postulate that $u_t$ is an AR(1) error so that $u_t = \rho u_{t-1} + \epsilon_t$ with $|\rho| < 1$ and $\epsilon_t$ being white noise with zero mean and finite variance. We shall refer to this model as the Box–Cox/AR(1) model.

Treating the Box–Cox/AR(1) model as the unrestricted model, we apply the likelihood ratio test to determine whether the data will reject the following restricted models: (1) double-log/AR(1): $\lambda = 0$; (2) linear/AR(1): $\lambda = 1$; (3) Box–Cox/white noise: $\rho = 0$; (4) double-log/white noise: $\lambda = 0$ and $\rho = 0$; and (5) linear/white noise: $\lambda = 1$ and $\rho = 0$.

*We have omitted the residential infra-marginal price as one of the regressors because of the lack of disaggregated data. The effect of this omission should be small, in view of the convincing argument put forth by Berndt (1990, ch. 7). Because the time-of-day binary variables are highly correlated with $S_t$ (see table 1), they are not included in the regression analysis.
Agthe et al. (1986) and Deller et al. (1986) argue that the average price, $P$, may be correlated with the error term, $u$. We perform the Hausman test by running an expanded regression for each model. This expanded regression includes an additional regressor, the price instrument constructed using a linear regression model.  

Table 3 reports the likelihood ratio test results which indicate that the data do not reject the double-log/AR(1) and linear/AR(1) models at the 1 percent level. The Hausman test results show that the data do not reject the null hypothesis of $P$ and $u$ being uncorrelated. Hence, the double-log/AR(1) and linear/AR(1) models are plausible specifications for explaining the per capita use of water in Hong Kong.

Table 4 presents the estimates for the double-log/AR(1) and linear/AR(1) models. Both models yield a good fit with adjusted $R^2$s over 0.9. While there is autocorrelation, the Durbin–Watson statistics show that the transformed residuals are serially uncorrelated. All coefficient estimates have the expected signs. Except for the rainfall variable and the intercept under the linear/AR(1) specification, all coefficient estimates are statistically significant at the 1 percent level.

The own-price and income elasticities based on the double-log/AR(1) specifications are respectively equal to -0.4684 and 0.2354, similar to those in Agthe et al. (1986), Martin and Thomas (1986), Deller et al. (1986) and Moncur (1987). The estimate for $(1-\phi)$ is 0.1642 with a standard error of 0.0301, indicating that the double-log/AR(1) specification is consistent with consumer maximization. The findings based on the linear/AR(1) specification are similar and are not repeated.

We apply eqs. (7)-(10) to compute the welfare losses for the following changes in supply hours: (1) from 24 to 20 hours per day; (2) from 24 to 16 hours per day; and (3) from 24 to 10 hours per day. Associated with these changes are the following daily unserved hours: 4, 8 and 14. Since eqs. (7), (9) and (10) are non-linear, we use sample enumeration to compute the per capita welfare loss ($/month).

Table 5 indicates that consumption reduction through service interruption

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9The dependent variable is $P$, and the independent variables include an intercept, $Y$, $S$, $W_1$, $W_2$, $D_1$, $D_2$, $D_3$, $Z_1$, $Z_2$, $Z_3$. See table 2 for the variable definitions.

10Because of limited land, residential irrigation of lawns and gardens is almost non-existent in Hong Kong. As a result, an increase in rainfall does not have a significant effect on water consumption.

11Since $(1-\phi)$ is the elasticity of consumption with respect to supply hours, we can use it to predict the impact of service interruption on water consumption. For example, a policy of 8 unserved hours per day would result in approximately 6.65 percent ($=0.1642 \times \ln(16/24)$) reduction in monthly use. The same reduction can be achieved by increasing the average rate by 13.68 percent ($= 6.65 \text{ percent}/0.4684$); see Moncur (1987) for a similar calculation.

12We first compute the welfare loss for each month and then take the average of the monthly results.
Table 3

<table>
<thead>
<tr>
<th>Model</th>
<th>λ</th>
<th>ρ</th>
<th>Durbin-Watson statistic</th>
<th>Log-likelihood</th>
<th>Likelihood ratio statistic</th>
<th>Degrees of freedom</th>
<th>Hausman test statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Box-Cox/AR(1)</td>
<td>0.58</td>
<td>0.800</td>
<td>2.0268</td>
<td>-24.88</td>
<td>N.A.</td>
<td>N.A.</td>
<td>0.3354</td>
</tr>
<tr>
<td>Double log/AR(1)</td>
<td>0.00</td>
<td>0.790</td>
<td>1.9840</td>
<td>-26.08</td>
<td>2.40</td>
<td>1</td>
<td>0.2043</td>
</tr>
<tr>
<td>Linear/AR(1)</td>
<td>1.00</td>
<td>0.803</td>
<td>2.0497</td>
<td>-25.48</td>
<td>1.20</td>
<td>1</td>
<td>0.4464</td>
</tr>
<tr>
<td>Box-Cox/white noise</td>
<td>0.91</td>
<td>0.000</td>
<td>0.8623</td>
<td>-66.29</td>
<td>82.82</td>
<td>1</td>
<td>0.5284</td>
</tr>
<tr>
<td>Double-log/white noise</td>
<td>0.00</td>
<td>0.000</td>
<td>0.7995</td>
<td>-69.11</td>
<td>88.46</td>
<td>2</td>
<td>0.7310</td>
</tr>
<tr>
<td>Linear/white noise</td>
<td>1.00</td>
<td>0.000</td>
<td>0.8677</td>
<td>-66.32</td>
<td>82.88</td>
<td>2</td>
<td>0.4907</td>
</tr>
</tbody>
</table>

*Likelihood ratio statistic = \(-2(\text{log-likelihood(restricted)} - \text{log-likelihood(unrestricted)})\) which is distributed as \(\chi^2\) with d.f. equal to the number of restrictions; \(\chi^2 = 6.635 \text{ with 1 d.f. at 1 percent level}; \chi^2 = 9.210 \text{ with 2 d.f. at 1 percent level}; \text{and N.A. = not applicable.}\)

*Hausman test statistic = standard normal variate (z) and \(z = 2.576 \text{ at 1 percent level.}\)
Table 4

Monthly per capita water consumption \( (Q) \) model sample period: April 1974–March 1984 (132 observations).

<table>
<thead>
<tr>
<th>Variable with expected sign in [ ]</th>
<th>Double-log/AR(1)</th>
<th>Linear/AR(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept ([?])</td>
<td>-5.2209*</td>
<td>-0.7730</td>
</tr>
<tr>
<td>( P_i ); monthly average rate (($/m^3)) ([-)]</td>
<td>-0.4684*</td>
<td>-9.0284*</td>
</tr>
<tr>
<td>( Y_i ); monthly per capita income (($)) ([+)]</td>
<td>0.2354*</td>
<td>0.0019*</td>
</tr>
<tr>
<td>( S_i ); monthly supply hours ([+)]</td>
<td>0.1642*</td>
<td>0.0076*</td>
</tr>
<tr>
<td>( W_{1i} ); (rainfall–evaporation): actual–normal ((\text{mm/month})) ([-)]</td>
<td>-0.000038</td>
<td>-0.0002</td>
</tr>
<tr>
<td>( W_{2i} ); average temperature: actual–normal ((^\circ\text{C})) ([+)]</td>
<td>0.0094*</td>
<td>0.0742*</td>
</tr>
<tr>
<td>( D_{1i} ); =1, if first quarter; (=0), otherwise ([?])</td>
<td>-0.0456*</td>
<td>-0.3135*</td>
</tr>
<tr>
<td>( D_{2i} ); =1, if second quarter; (=0), otherwise ([?])</td>
<td>0.0502*</td>
<td>0.3366*</td>
</tr>
<tr>
<td>( D_{3i} ); =1, if third quarter; (=0), otherwise ([?])</td>
<td>0.0498*</td>
<td>0.3632*</td>
</tr>
<tr>
<td>( N_i ); number of calendar days per month ([+)]</td>
<td>1.2010*</td>
<td>0.2700*</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.7897*</td>
<td>0.8025*</td>
</tr>
<tr>
<td>Adjusted ( R^2 )</td>
<td>0.9143</td>
<td>0.9150</td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>-26.0761</td>
<td>-25.4755</td>
</tr>
<tr>
<td>Durbin–Watson statistic</td>
<td>1.9840</td>
<td>2.0497</td>
</tr>
<tr>
<td>Standard error of regression</td>
<td>0.0384</td>
<td>0.2923</td>
</tr>
</tbody>
</table>

\( ^* \text{Significant at 1 percent level.} \)
\( ^a \)Coefficient estimate for log (variable).

Note: Standard errors in parentheses.

creates large welfare losses.\(^{13}\) For example, the per capita \( CV_S \) estimate under the double/AR(1) specification ranges from $221 to $1,607 per month. The per capita \( CV_S \) estimate is increasing in interruption duration at an increasing rate. The per capita \( CV_S \) estimate under the linear/AR(1) specification is proportional to the monthly interruption duration and is smaller than the one under the double-log/AR(1) specification.

Three factors account for the large \( CV_S \) estimates. First, service interruption is assumed to occur daily, implying a large number of unserved hours per month. Even though the estimated per capita welfare loss per hour unserved appears to be reasonable ($1.36 to $3.8/hour), the estimated total loss is large. Second, service interruption only allows consumption during the

\(^{13}\)Numerical results indicate that both conditions C.2 and C.3 are satisfied for all 132 monthly observations.
Table 5

Average monthly per capita welfare loss within-sample simulation (132 observations). Number of calendar days per month = 30.44; see table 2.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Double-log/AR(1)</th>
<th>Linear/AR(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total unserved hours per month</td>
<td>121.76</td>
<td>121.76</td>
</tr>
<tr>
<td>Total unserved water (m³/month)</td>
<td>0.2293</td>
<td>0.3127</td>
</tr>
<tr>
<td>CVₗ, welfare loss (interruption) ($/month)</td>
<td>221.12</td>
<td>166.03</td>
</tr>
<tr>
<td>CVₗ, per unserved hour ($/hour)</td>
<td>1.816</td>
<td>1.364</td>
</tr>
<tr>
<td>VP, virtual price ($/m³)</td>
<td>0.4012</td>
<td>0.4117</td>
</tr>
<tr>
<td>CVₘ, welfare loss (pricing) ($/month)</td>
<td>0.1783</td>
<td>0.2686</td>
</tr>
</tbody>
</table>

a4 unserved hours per day.
b8 unserved hours per day.
*c14 unserved hours per day.

supply hours, thus severely limiting a consumer's choice set. In contrast, a quantity constraint allows the consumer to allocate the total monthly consumption among the hours of the month; see eq. (1). Finally, our parametric specification of $Q(P, Y, S)$ may be restrictive in the determination of $CV_s$. For instance, the linear specification implies $CV_s(P, Y, S_0, S_1) = (1-\phi)(S_0-S_1)/\beta$ and $Q(P, Y + CV_s, S_1) = Q(P, Y, S_0)$. Thus, maintaining the utility level at $V(P, Y, S_0)$ requires an income increase that will keep consumption unchanged. Since $Q(P, Y, S)$ is income inelastic, the resulting $CV_s$ is large.

The per capita $CV_p$ estimate is less than $1 per month, indicating that the service interruption strategy is highly inefficient relative to pricing in reducing water consumption. The $CV_s$ estimate exceeds the $CV_p$ estimate by more than 500 times. This finding of service interruption being inefficient is insensitive to the choice of functional form.

5. Conclusion

In this paper we have specified a model of consumer behaviour under service interruption and implemented it using water consumption data. From this model we have developed the exact welfare loss of water service interruption designed to reduce consumption during times of severe drought. Using the same model, we have also computed the welfare loss due to a price increase that yields the same amount of consumption reduction. Since our welfare loss estimates are based on actual market data, they are free from the common criticisms related to the results developed from contingent valuation
survey data. The major finding is that the welfare loss of water service interruption greatly exceeds that of a price increase, indicating that service interruption is very inefficient for water shortage management.

Appendix: Data description and sources

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_t$</td>
<td>Monthly per capita water use ($m^3$) = (monthly water consumption/monthly population). Monthly population is estimated by linear interpolation using mid-year estimates. Source: Hong Kong Monthly Digest of Statistics, various years.</td>
</tr>
<tr>
<td>$P_t$</td>
<td>Monthly average rate ($$/m^3$) = (Water Department's fiscal year revenue/Fiscal year water consumption), deflated by monthly CPI (April 1973 = 1.00). Source: Hong Kong Annual Report and Hong Kong Monthly Digest of Statistics, various years</td>
</tr>
<tr>
<td>$S_t$</td>
<td>Monthly supply hours = monthly total hours - monthly total unserved hours. Source: Table 1.</td>
</tr>
<tr>
<td>$W_{1t}$</td>
<td>(Rainfall - evaporation): actual - normal (mm/month) = (monthly total rainfall - 30-year average of monthly rainfall) - (monthly total evaporation - 30-year average of monthly evaporation). Source: Hong Kong Monthly Digest of Statistics and Hong Kong Annual Report, various years.</td>
</tr>
<tr>
<td>$W_{2t}$</td>
<td>Average temperature: actual - normal (°C) = (monthly total temperature - 30-year average of monthly temperature). Source: Hong Kong Monthly Digest of Statistics and Hong Kong Annual Report, various years.</td>
</tr>
</tbody>
</table>

References

Bohn, R.E. et al. 1984, Optimal pricing in electricity network over space and time, Rand Journal of Economics 15, no. 3, 360-373.


DWR, 1987, California water: Looking to the future, Publication no. 160-87, California Department of Water Resources.


