Handout 5
Rent for the Firm

A. Welfare analysis for the production sector is much simpler than for consumption because of the absence of income effects (assuming that risk is not a concern).

B. Using the same notation that we have employed before, we can define profit, quasi-rent (R), and producer surplus (P) in a situation where some factors of production are fixed.

\[ \pi = p \cdot y - TFC; \quad R = P = p \cdot y. \]

\( \Delta R \) is the desired measure of welfare change caused by a change in price. If price changes in a way that does not cause the firm to cease production, then \( \Delta R = \Delta \pi \) and \( \Delta \pi \) is an okay measure. If, however, the price change causes a halt in production, \( \Delta R \neq \Delta \pi \), because the firm not only loses its profits – it still incurs fixed costs. \( \Delta R \) remains a correct measure.

C. Assuming that technology is captured by \( f(y) \leq 0 \), profit-maximization conditions are, as before,

\[ p_n = \delta \cdot \frac{\partial f}{\partial y_n} \quad \text{for all } n. \]

These conditions can be used to determine the firm's optimal excess supply functions:

\[ y_n^* = y_n^*(p). \]

So, \( \pi = p \cdot y^*(p) - TFC \) and \( R(p) = p \cdot y^*(p) \).

Taking the derivative of \( R \) with respect to an arbitrary price,

\[ \frac{\partial R}{\partial p_n} = y_n^*(p) + p \cdot D_n y_n^*(p) \]

where \( D_n \) is the \( N \rightarrow N \) differential operator.

Equivalently, \( \frac{\partial R}{\partial p_n} = y_n^*(p) + \sum_i p_i \frac{\partial y_i^*}{\partial p_n} \).

The final term of this expression is equal to zero according to the following proof. Differentiating \( f(y) = 0 \) totally, we obtain

\[ \sum_i \left( \frac{\partial f}{\partial y_i} \right) \left( \frac{\partial y_i}{\partial p_n} \right) = 0. \]

Substituting from [1],
$$\sum \left( \frac{p_i}{\delta} \right) \left( \frac{\partial y_i}{\partial p_n} \right) = 0,$$

or

$$\delta^{-1} \sum p_i \frac{\partial y_i}{\partial p_n} = 0$$

Hence, \( \frac{\partial R}{\partial p_n} = y_n^*(p) \). \text{[Hotelling's Lemma]} \hspace{2cm} (2)

D. Suppose the initial price vector is \( p^0 \) and the subsequent price vector is \( p^1 \). Then, (no fixed costs here)

$$\Delta R = R\left( p^1 \right) - R\left( p^0 \right) = \pi^1 - \pi^0.$$ \hspace{2cm} (3)

Equation [3] can be used to calculate the change in quasi-rent if you have complete knowledge on how prices affect profits. Usually, however, this information cannot be directly observed, and an alternative method is popular:

E. \( \Delta R = R\left( p^1 \right) - R\left( p^0 \right) = \int_L dR \)

\[= \int_L \left( \sum_n y_n^* (p) dp_n \right) \]

\[= \sum_n \int p_n^{1} y_n^* (p) dp_n. \] \hspace{2cm} (4)

Equation [4] has chosen a particular path of integration (L), but the path is unimportant because of path independence. "Path" refers to a selected sequence of price changes.

F. \( \Delta R \) for an output price change (\( p_n \))

It is important to observe a crucial assumption in all the following formulae of this handout. It is assumed that no other prices change in response to the analyzed price change(s). That is, a partial equilibrium situation is assumed. Some authors handle this by assuming that all other supplies and demands are perfectly elastic.

1. Measured in the output market (Figure 1.1) where \( p_n^{\min} \) indicates \( p_n \) at which marginal cost curve intersects average variable costs.
R^0 = \int_{p^{\text{min}}_n}^{p^{\text{max}}_n} y^*_n dp_n = a + b
\nR^1 = \int_{p^{\text{min}}_n}^{p^{\text{max}}_n} y^*_n dp_n = b
\n\Delta R = \int_{p^{\text{min}}_n}^{p^{\text{max}}_n} y^*_n dp_n
\nonlyear\n\Delta R = R^1 - R^0 = -a

2. Measured in an input \((y_i)\) market (Figure 1.2)

Full measurement of an output price change in an input market requires that the input be essential (Just, Hueth, Schmitz, p. 63). An input is essential if zero input usage implies zero output.

R^0 = \int_{\bar{p}_i}^{p^{\text{max}}_i} \left(-y^*_i\left(p^0_i\right)\right) dp_i = a + b
\nR^1 = \int_{\bar{p}_i}^{p^{\text{max}}_i} \left(-y^*_i\left(p^1_i\right)\right) dp_i = b
\n\Delta R = \int_{\bar{p}_i}^{p^{\text{max}}_i} \left[-y^*_i\left(p^1_i\right) - \left(-y^*_i\left(p^0_i\right)\right)\right] dp_i
\nonlyear\n\Delta R = R^1 - R^0 = -a

G. \(\Delta R\) for an input price change \((p_i)\)

1. Measured in the output market (Figure 2.1)

R^0 = \int_{p^{\text{min}}_n}^{p^{\text{max}}_n} y^*_n(p^0) dp_n = a
\nR^1 = \int_{p^{\text{min}}_n}^{p^{\text{max}}_n} y^*_n(p^1) dp_n = a + b
\n\Delta R = \int_{p^{\text{min}}_n}^{p^{\text{max}}_n} \left[ y^*_n(p^1) - y^*_n(p^0) \right] dp_n
\nonlyear\n\Delta R = R^1 - R^0 = b

2. Measured in the input market (Figure 2.2)
\[ R^0 = \int_{p_i^0}^{p_i^{\text{max}}} (-y_i^*) dp_i = a \]

\[ R^1 = \int_{p_i^0}^{p_i^{\text{max}}} (-y_i^*) dp_i = a + b \]

\[ \Delta R = \int_{p_i^0}^{p_i^1} (-y_i^*) dp_i = R^1 - R^0 = b \]

H. \( \Delta R \) for multiple price changes

1. Sequential measurement is always possible but must be performed carefully.

2. Quasi-rent can be measured in any market (as always). Referring to Figure 3 as an example for an output market:

\[ R^0 = \int_{p_n^0}^{p_n^{\text{max}}} y_n^*(p_n^0) dp_n = a + b \]

\[ R^1 = \int_{p_n^0}^{p_n^{\text{max}}} y_n^*(p_n^1) dp_n = a + d \]

\[ \Delta R = R^1 - R^0 = d - b \]

References


Figure 3